

A Fast Immersed Boundary Method for Highly Deformable Elastic Bodies

By:

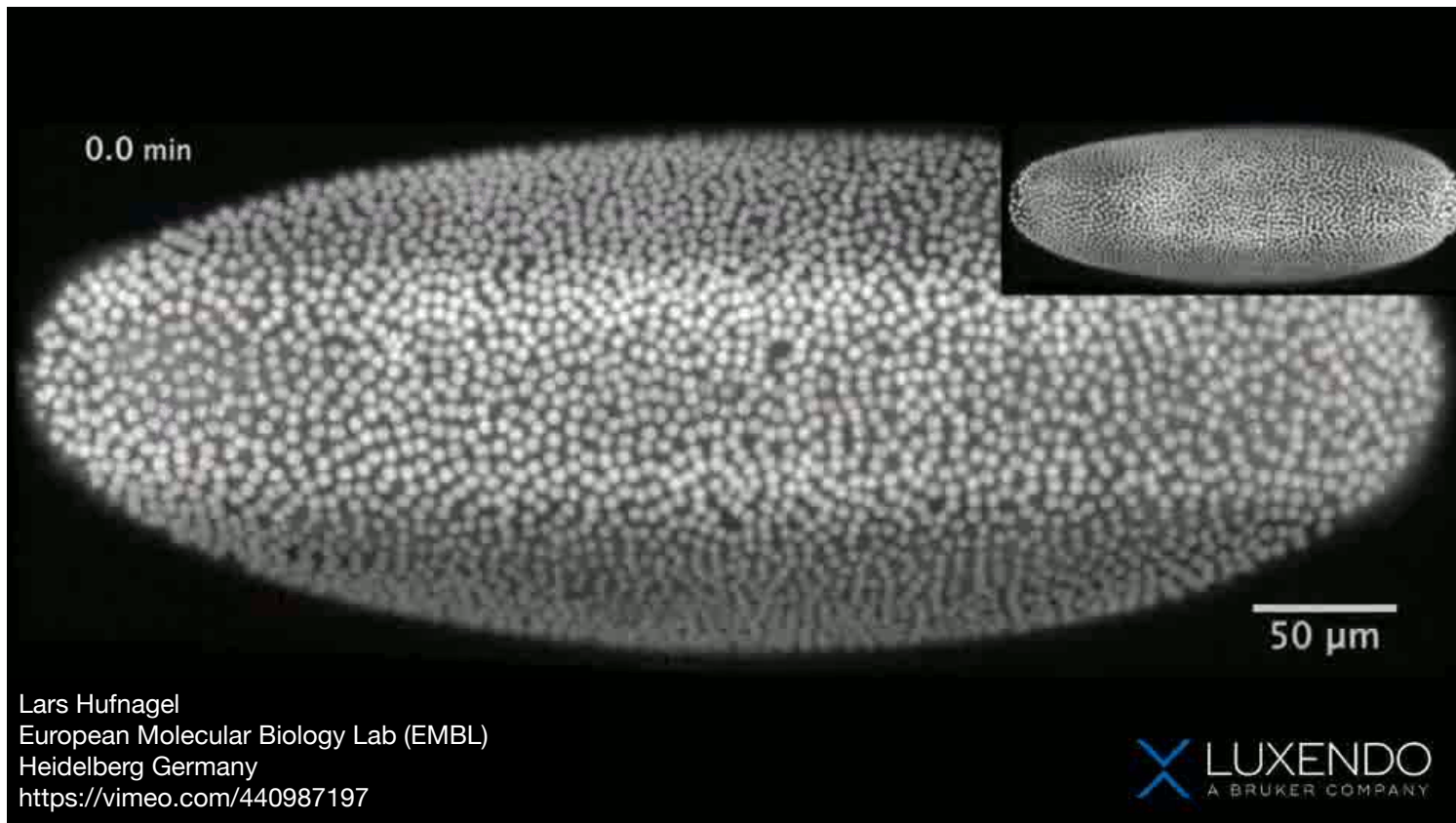
Mohamad Ibrahim Cheikh

Dobrovinski Lab

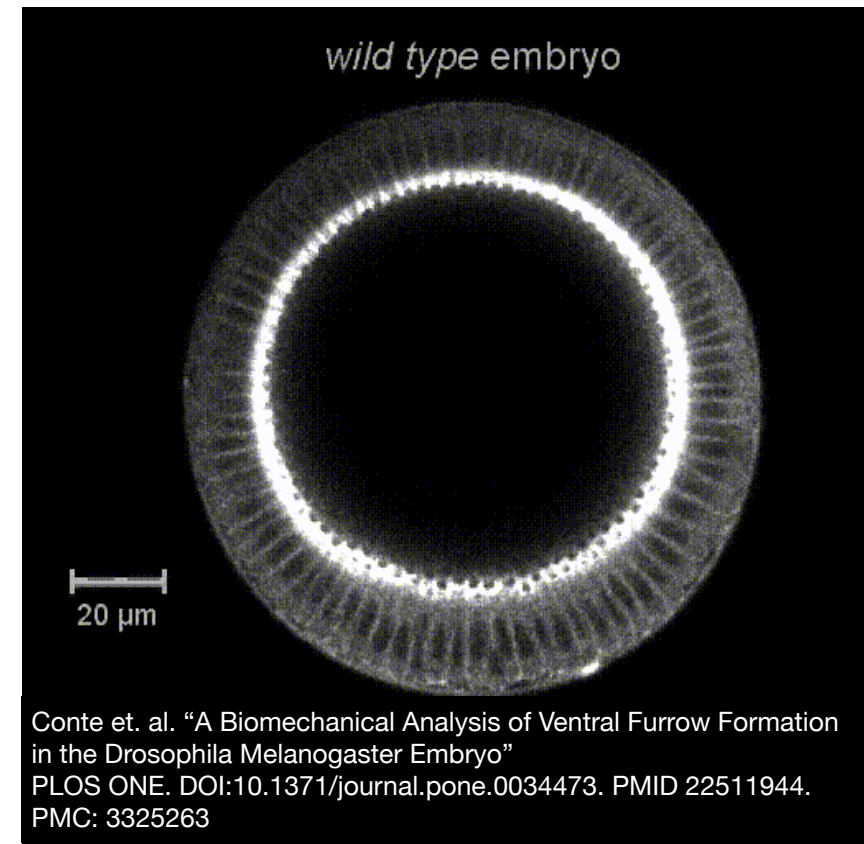
- Gastrulation

"It is not birth, marriage, or death, but *gastrulation*, which is truly the most important time in your life."

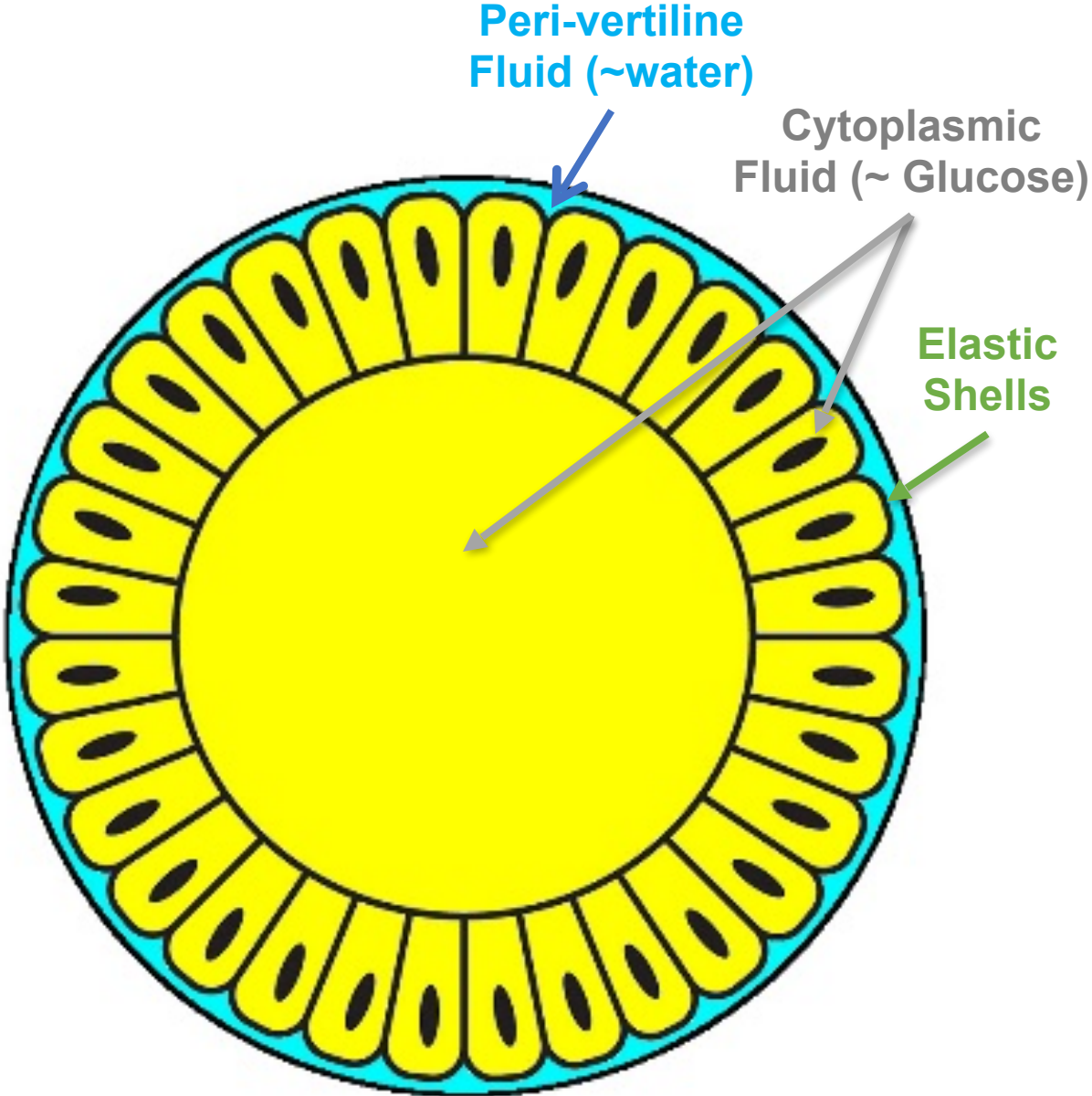
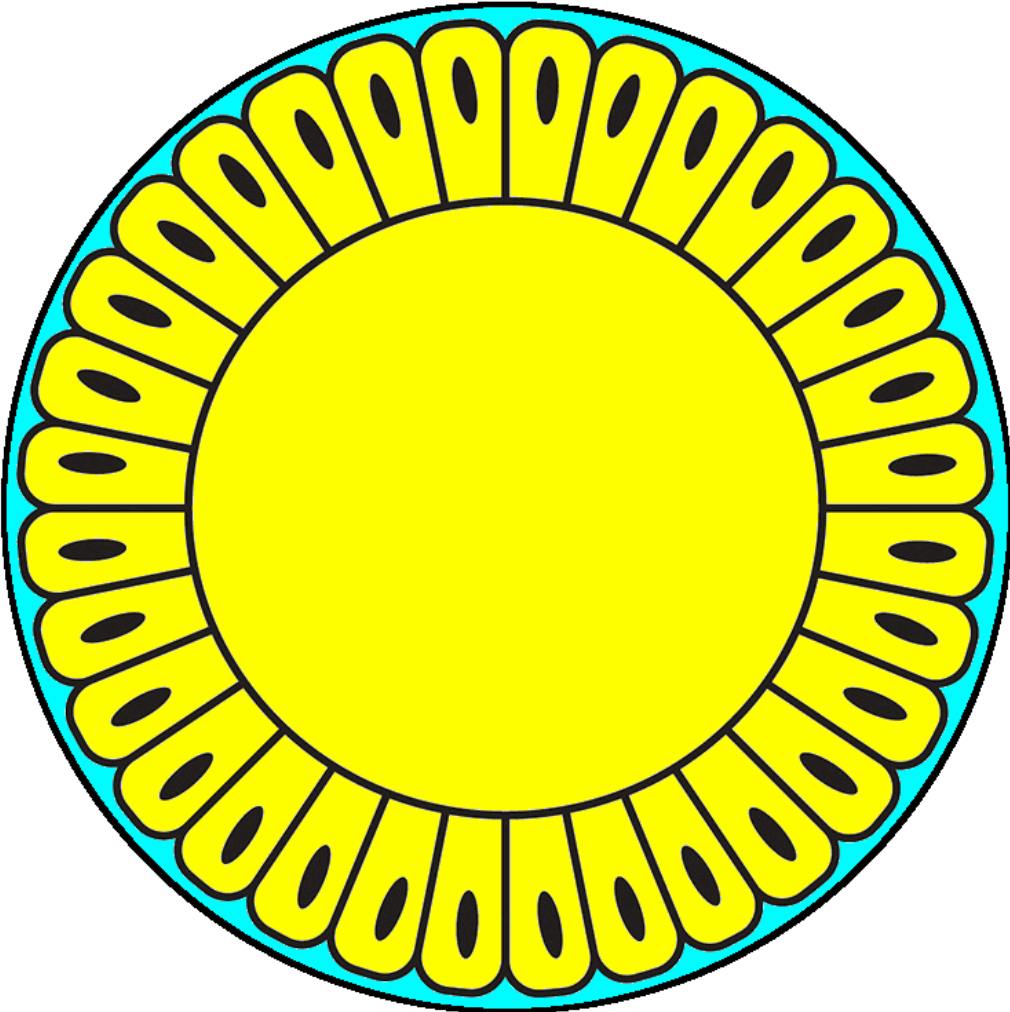
Lewis Wolpert (1986)



Ventral Furrow (VF) formation

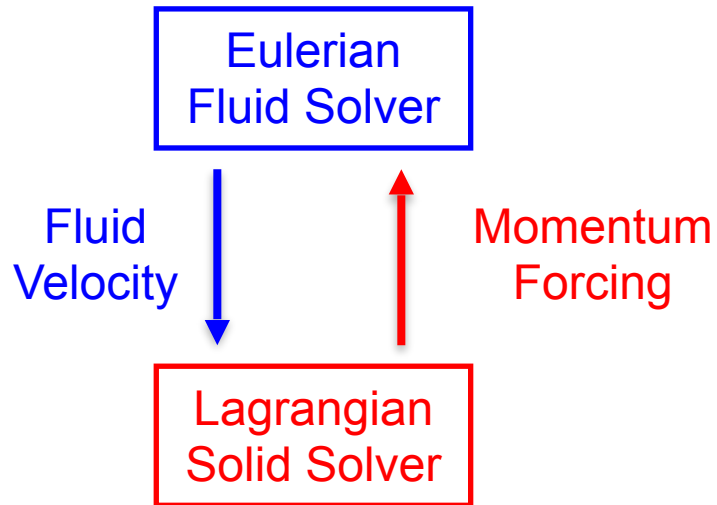


- Gastrulation



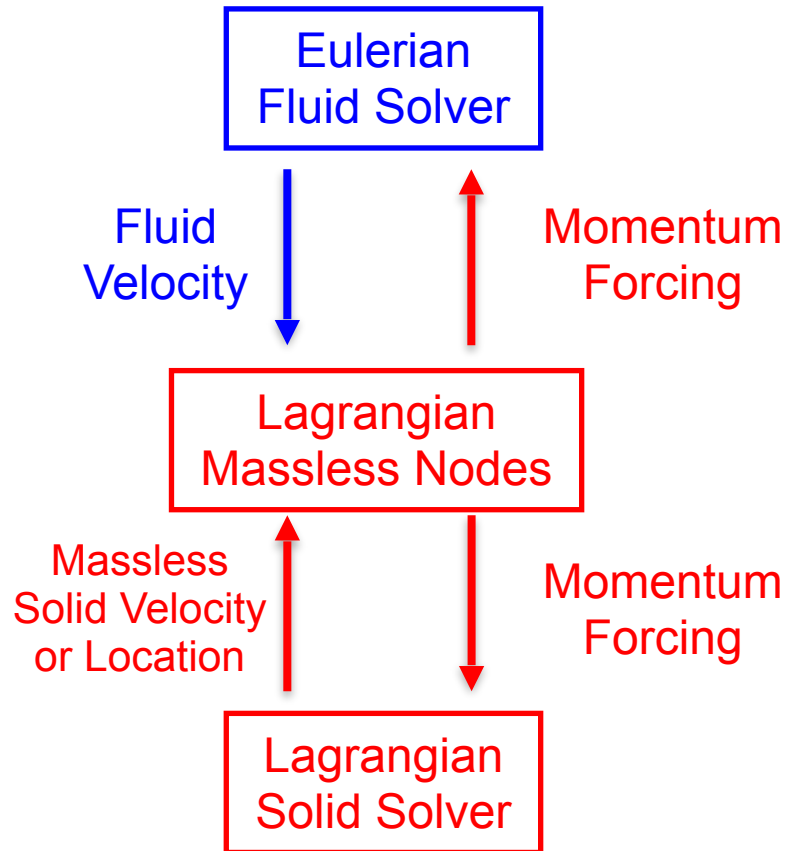
- Immersed Boundary Method

Direct Momentum Forcing:

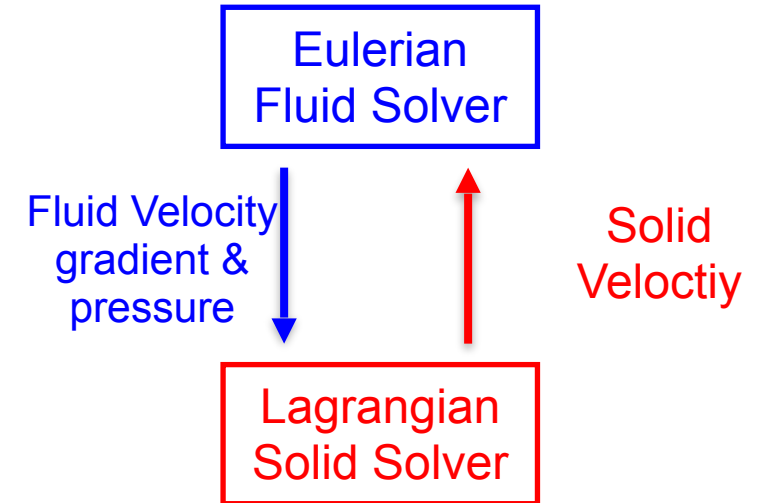


- Solid Massless Points

Direct Momentum Forcing
With Feedback:



Velocity Reconstruction:



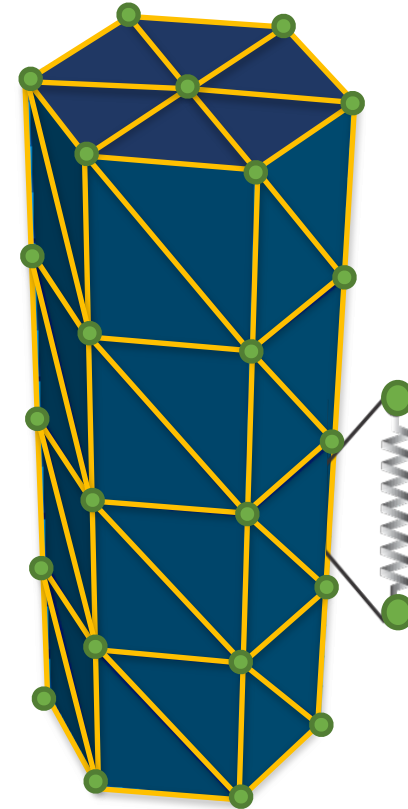
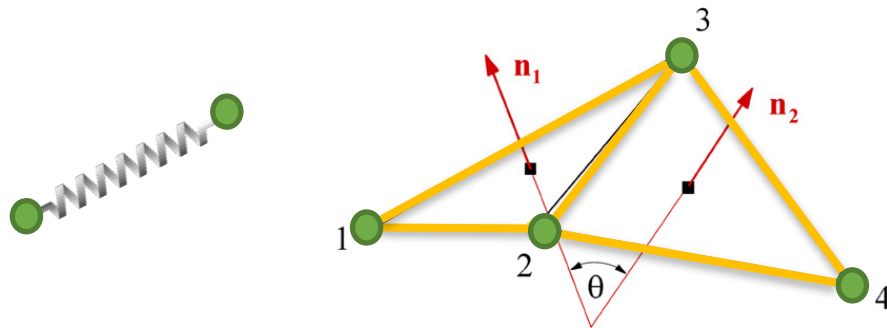
- Governing Laws - Solid

Mass-Spring system

$$m_s \ddot{\mathbf{X}}_s = \mathbf{F}_s \rightarrow \begin{cases} \dot{\mathbf{X}}_s = \mathbf{u}_f \\ \mathbf{F}_s \sim \mathbf{f}_f \end{cases}$$

Sum of external forces
(i.e., hydrodynamic loads,
gravity forces)

Sum of internal forces
(i.e. elastic forces)



Linear Elasticity

$$\mathbf{F}_e = k\mathbf{x}$$

- Governing Laws - Fluid

Navier-Stokes Equations:

$$\nabla \cdot \mathbf{u}_f = 0,$$

$$\rho_f \left(\frac{\partial \mathbf{u}_f}{\partial t} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f \right) = \nabla \cdot \left(\mu_f \left(\nabla \mathbf{u}_f + [\nabla \mathbf{u}_f]^T \right) \right) - \nabla p_f + \mathbf{f}_f$$

Dimensionless Number $RE = \rho_f UL / \mu_f \ll 10^{-8}$

Stokes Equations:

$$\nabla \cdot \mathbf{u}_f = 0,$$

$$0 = \nabla \cdot \left(\mu_f \left(\nabla \mathbf{u}_f + [\nabla \mathbf{u}_f]^T \right) \right) - \nabla p_f + \mathbf{f}_f$$

or

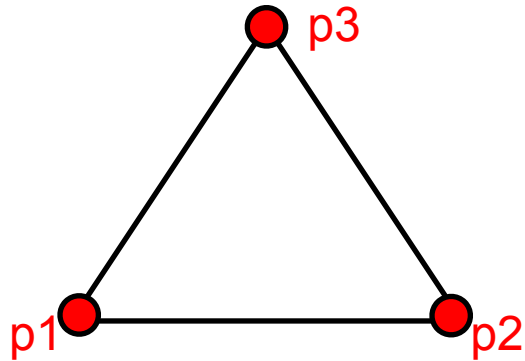
$$\rho_f \frac{\partial \mathbf{u}_f}{\partial t} = \nabla \cdot \left(\mu_f \left(\nabla \mathbf{u}_f + [\nabla \mathbf{u}_f]^T \right) \right) - \nabla p_f + \mathbf{f}_f$$

Numerical Discretization - Fluid

Define bases & Elements

Pressure:

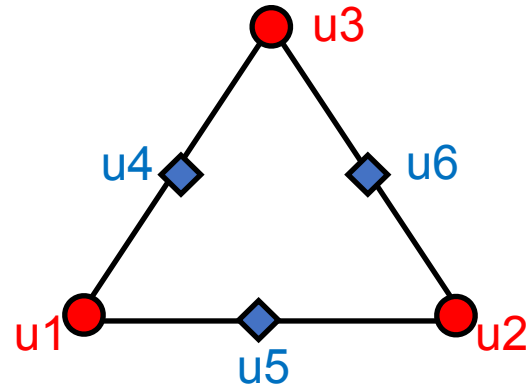
P1 Linear Element



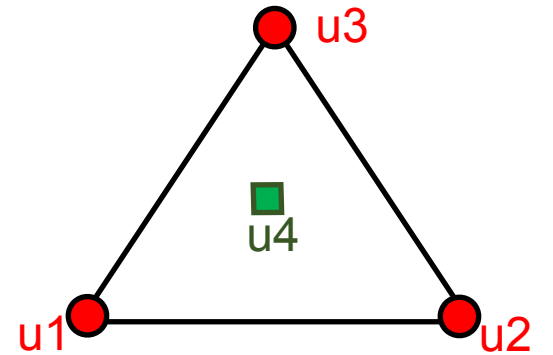
2D

Velocity:

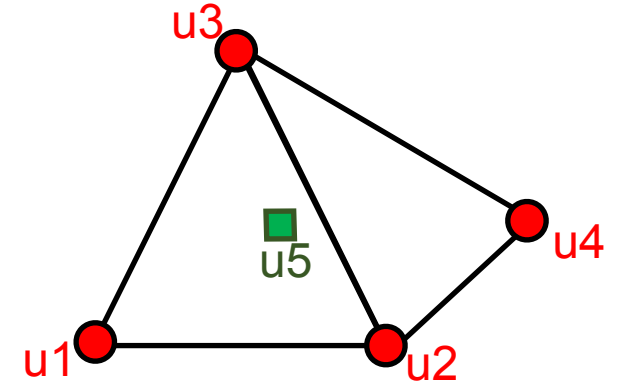
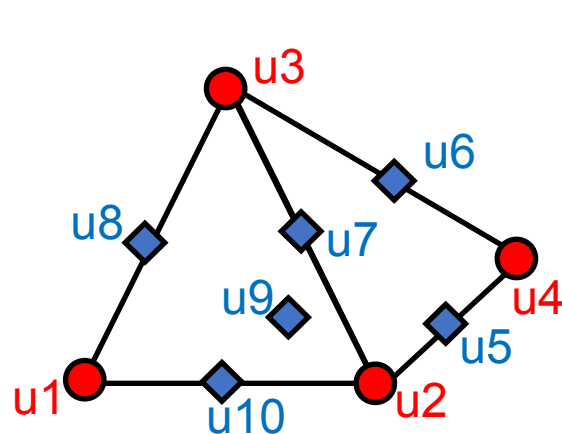
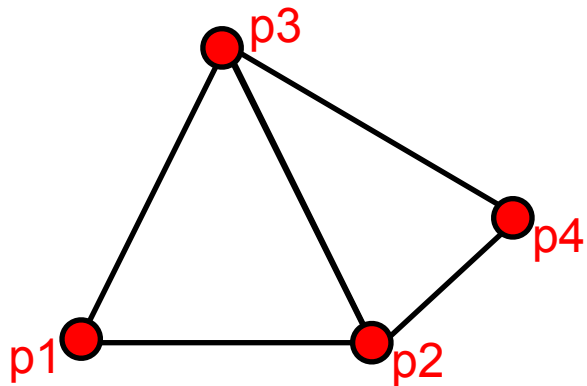
P2 Quadratic Element



P1b Cubic Element



3D

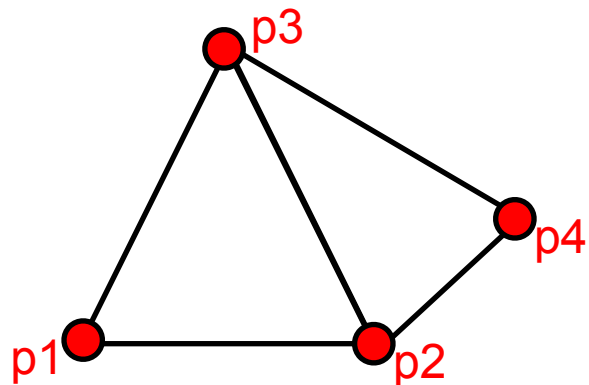
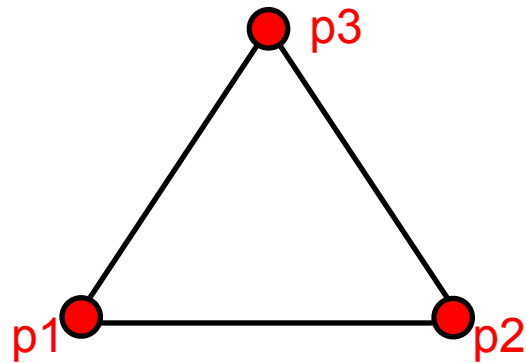


Numerical Discretization - Fluid

Define bases & Elements

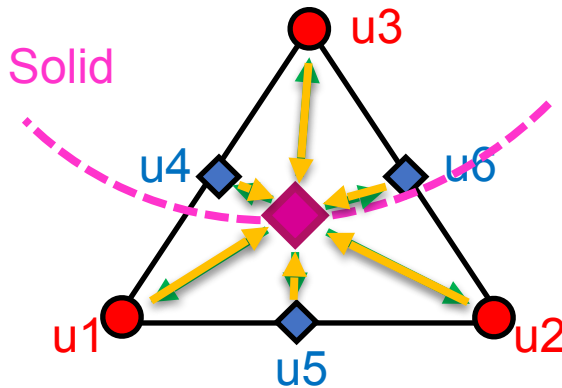
Pressure:

P1 Linear Element



Velocity:

P2 Quadratic Element



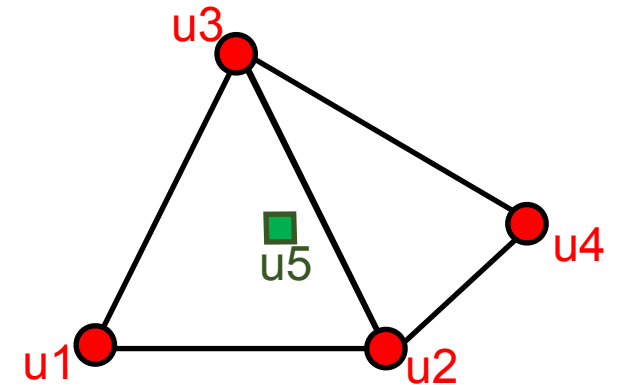
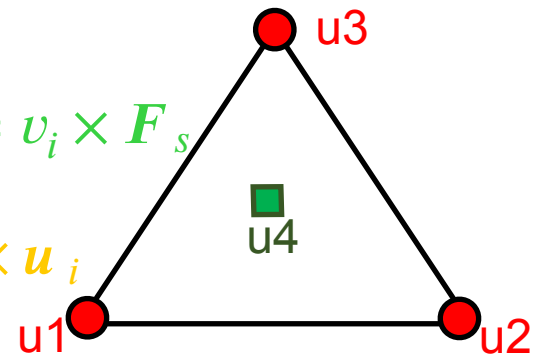
Fluid Forces:

$$\langle v, f_{f,i} \rangle = v_i \times F_s$$

Solid Velocity:

$$\dot{X}_s = \sum_i v_i \times u_i$$

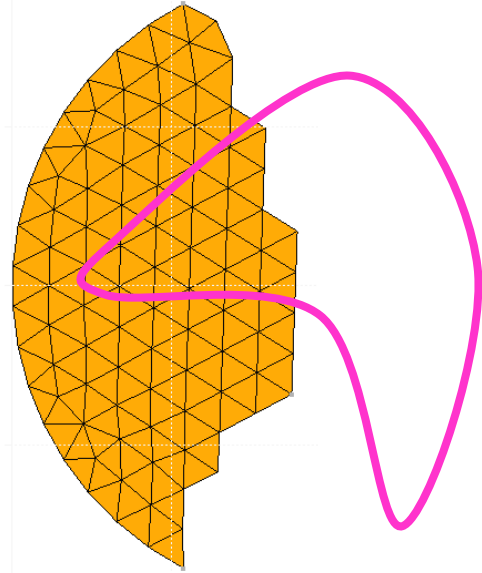
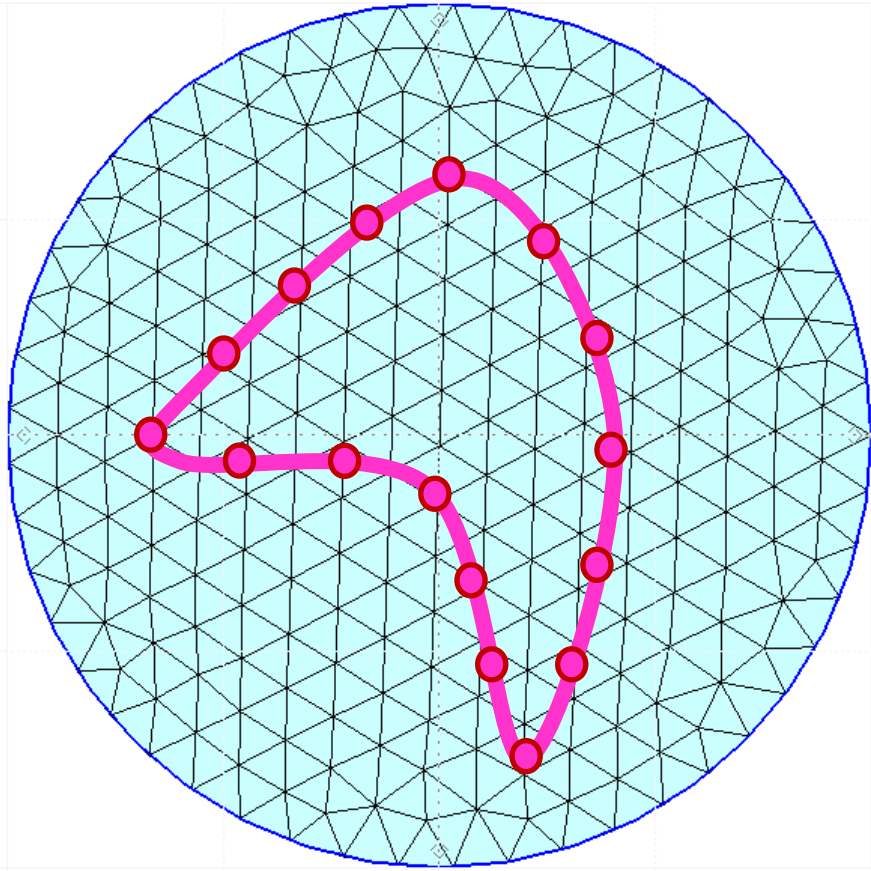
P1b Cubic Element



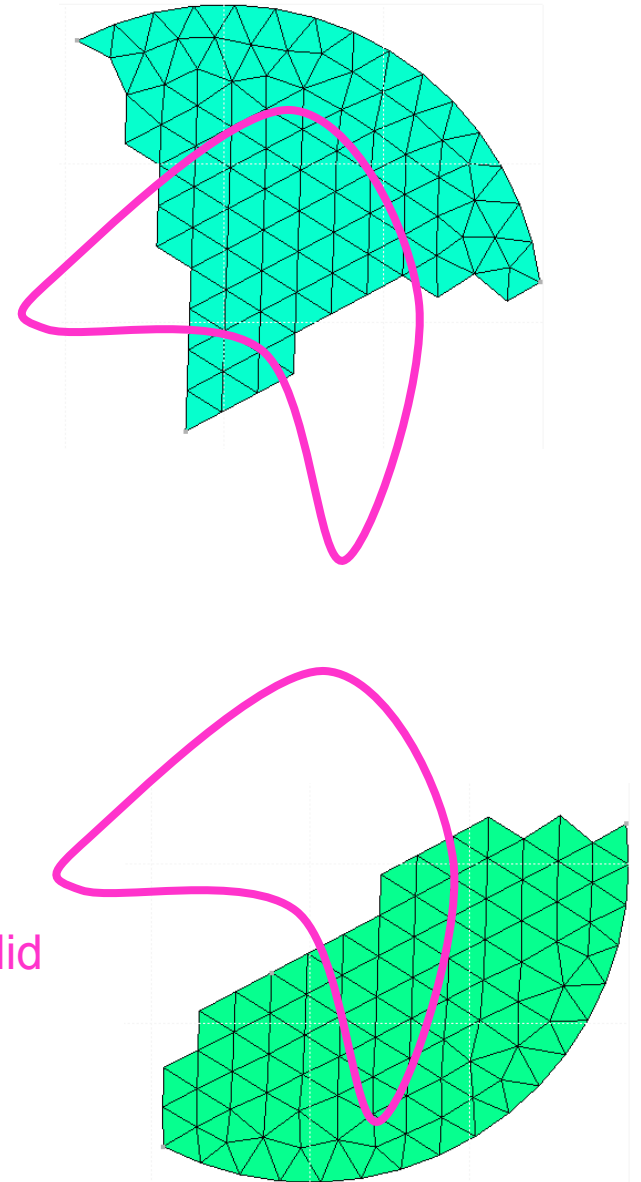
2D

3D

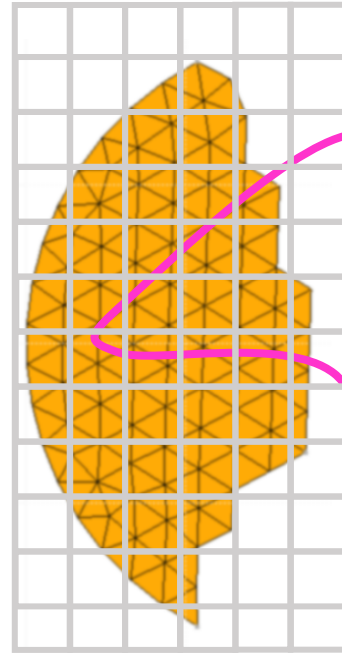
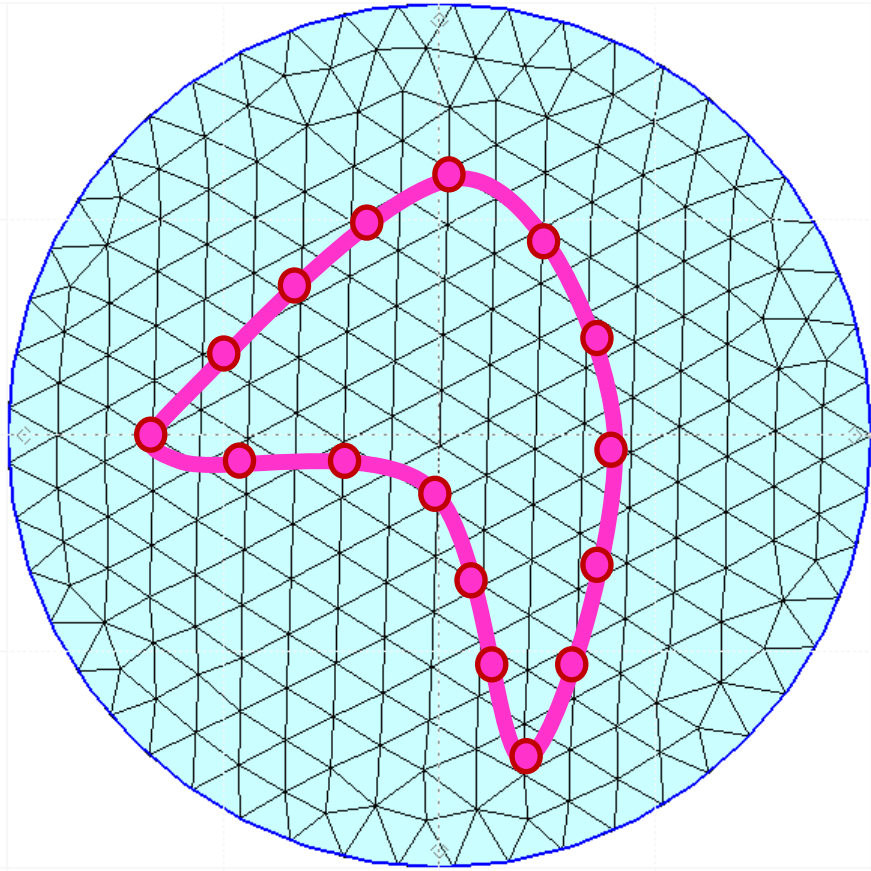
Immersed Boundary Method



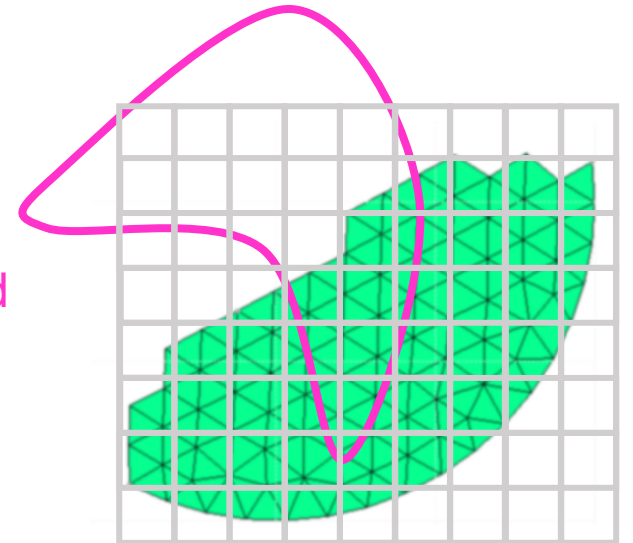
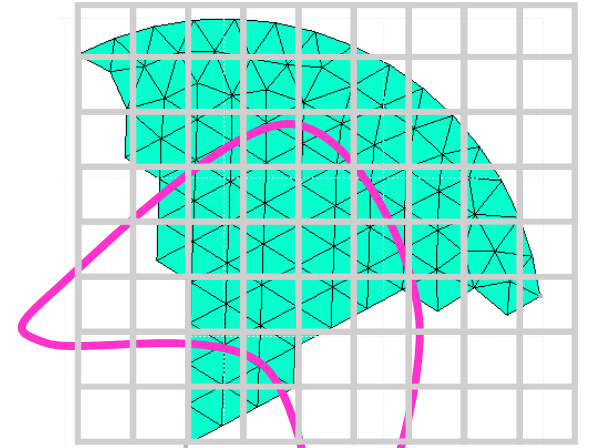
- Divide Eulerian Fluid
- Broadcast Lagrangian Solid



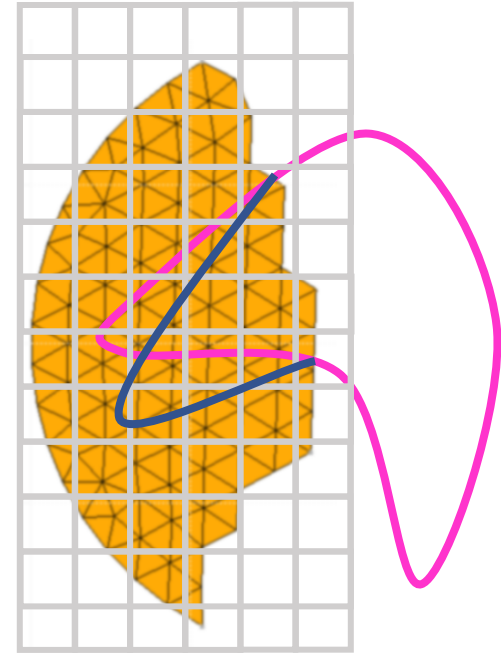
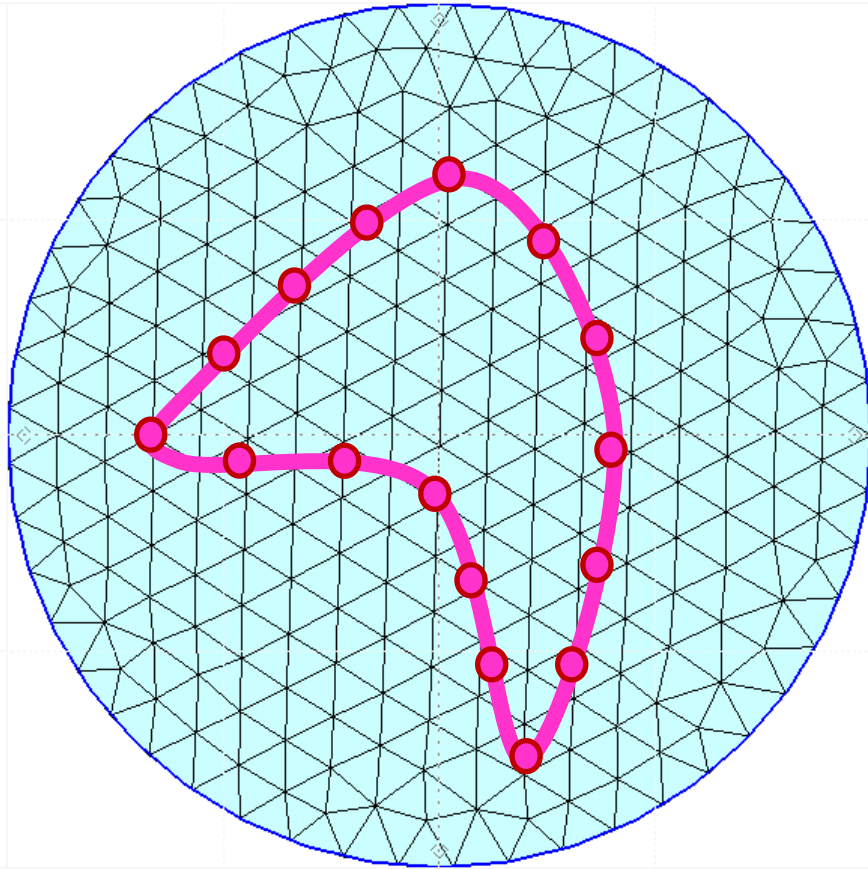
Immersed Boundary Method



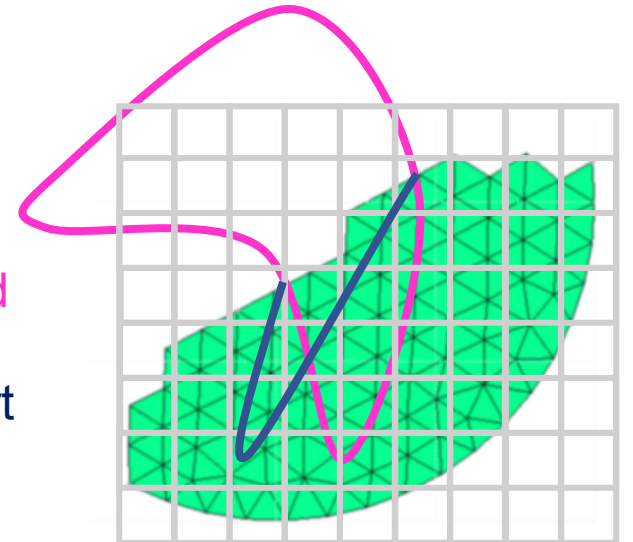
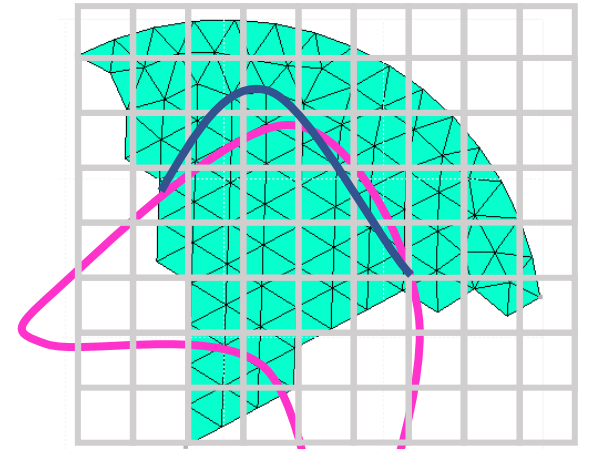
- Divide Eulerian Fluid
- Broadcast Lagrangian Solid
- Add Cartesian Boxes



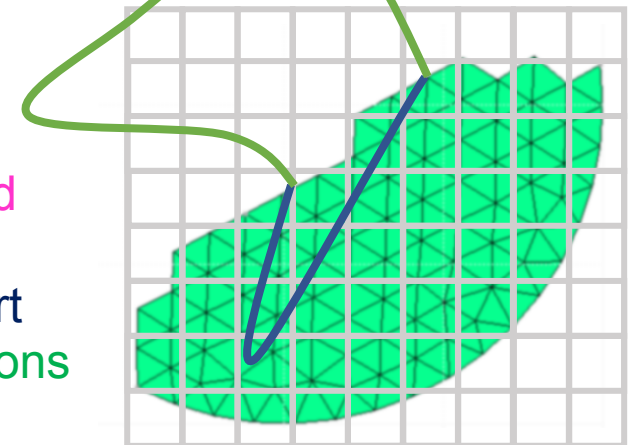
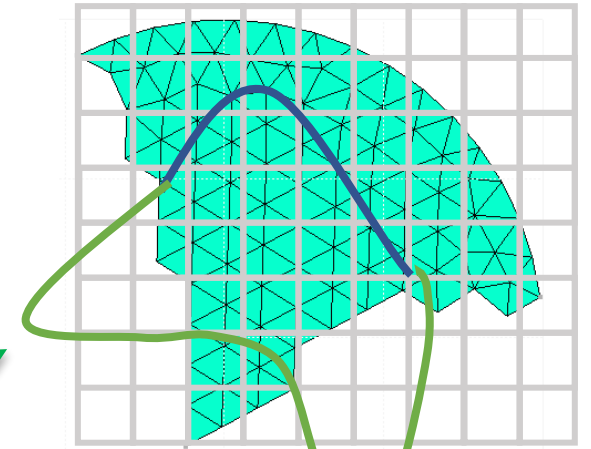
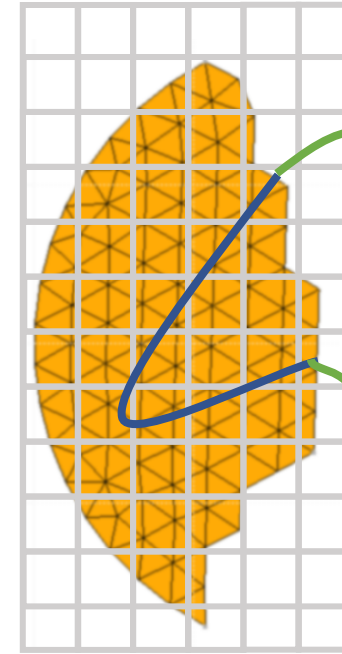
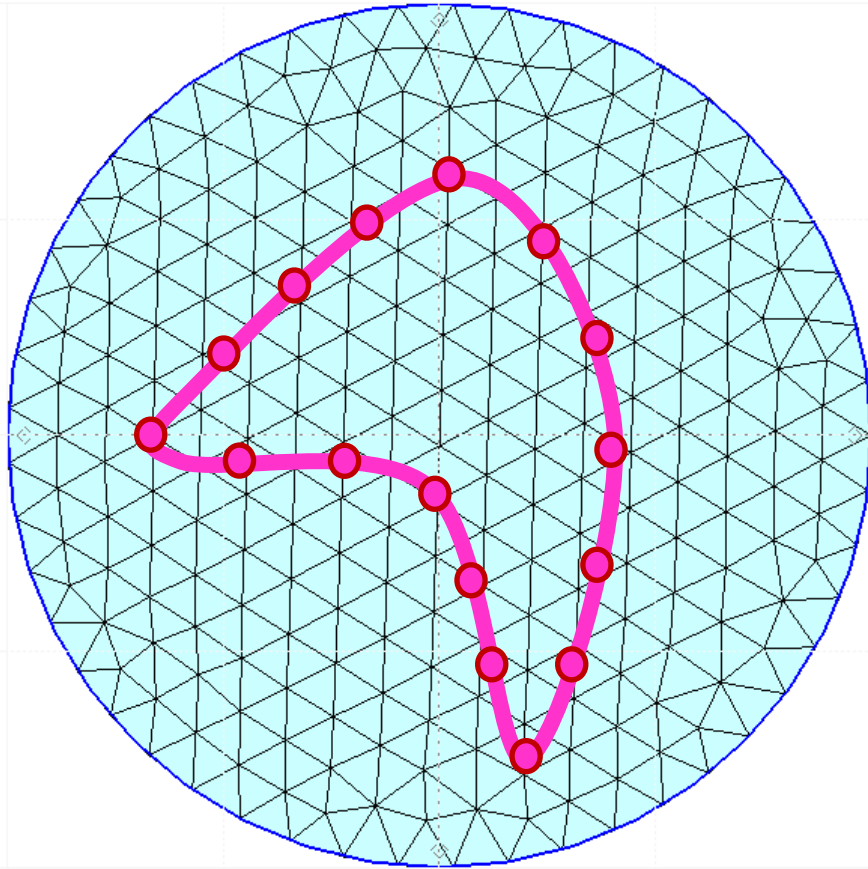
Immersed Boundary Method



- Divide Eulerian Fluid
- Broadcast Lagrangian Solid
- Add Cartesian Boxes
- Each Section Moves its part

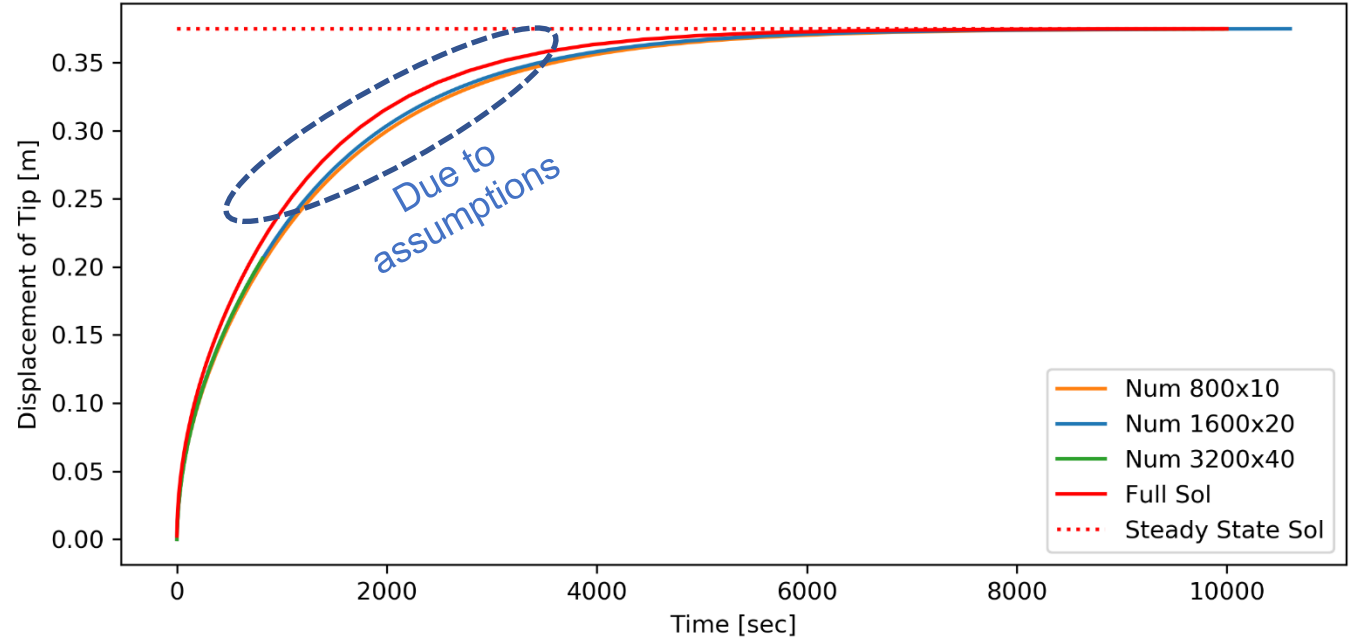
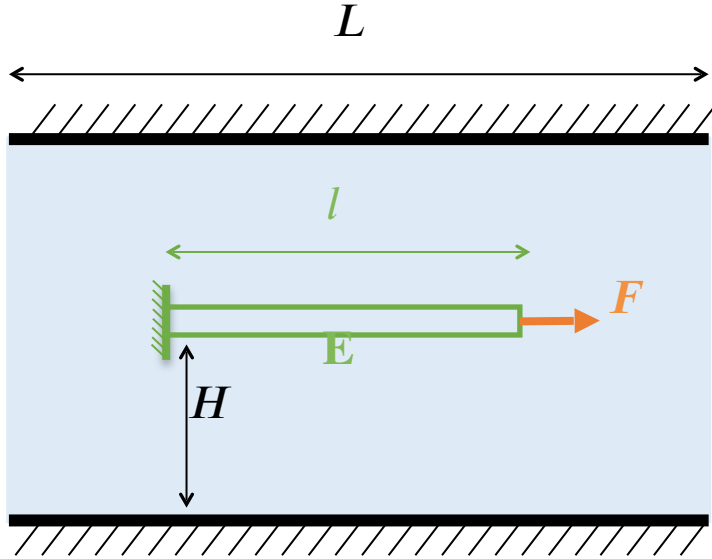


Immersed Boundary Method



- Divide Eulerian Fluid
- Broadcast Lagrangian Solid
- Add Cartesian Boxes
- Each Section Moves its part
- Broadcast the moved sections to all other nodes

Verification Case



Governing Equations:

$$2\mu \frac{\partial}{\partial y} \left(\frac{\partial \delta(x,t)}{\partial t} \right) = E \frac{\partial^2 \delta(x,t)}{\partial x^2}$$

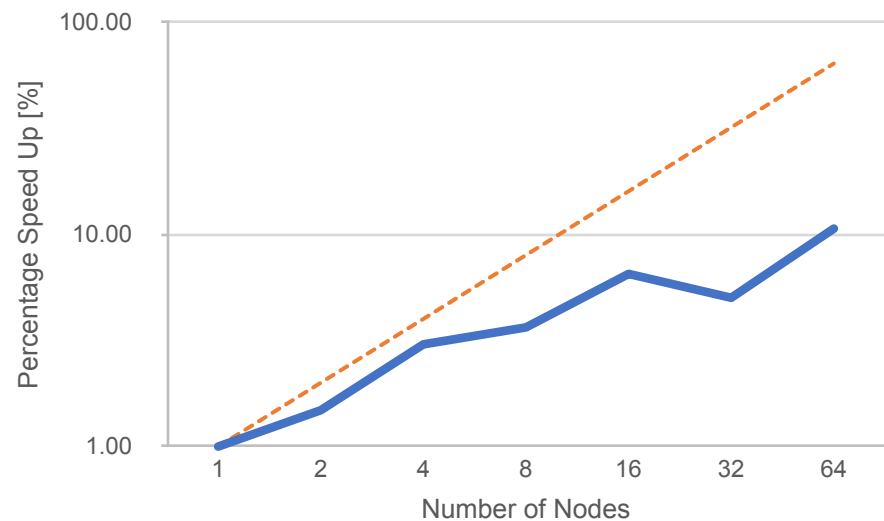
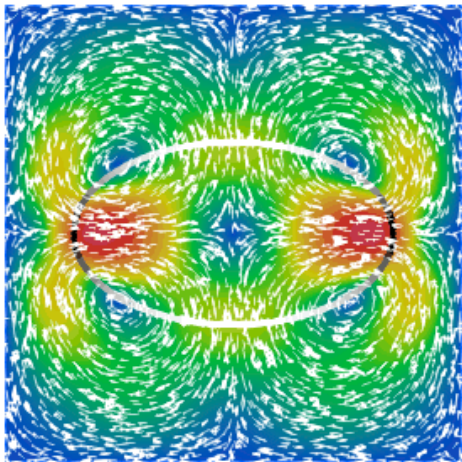
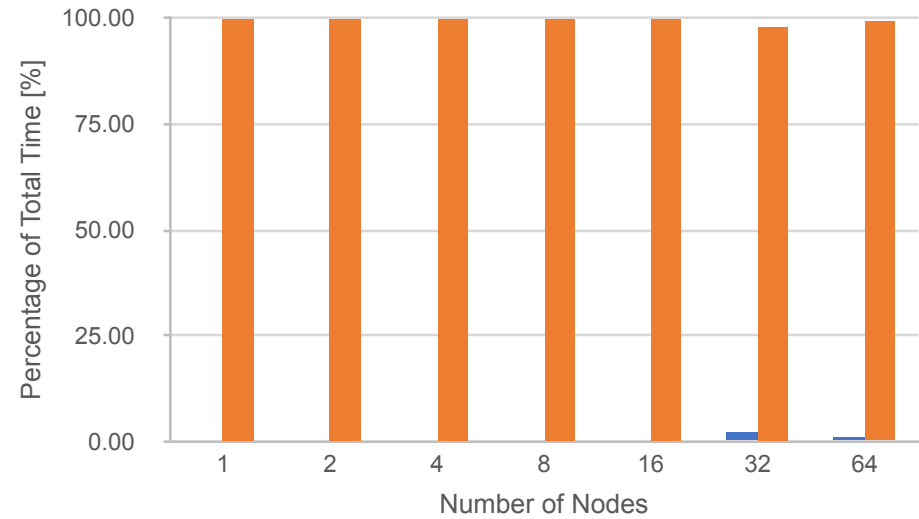
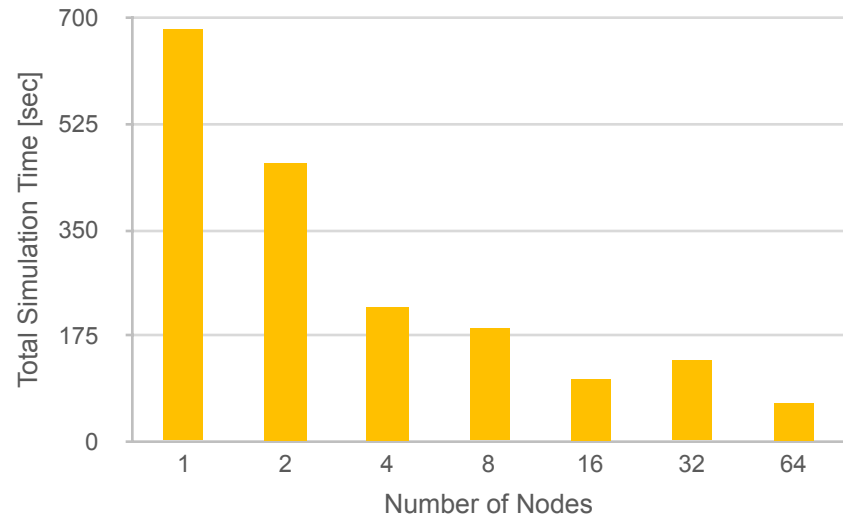
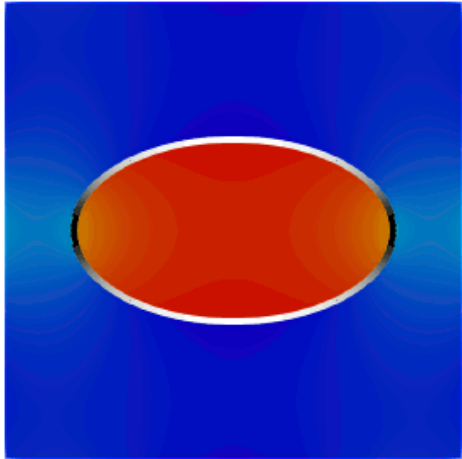
Assumption:

$H \ll l \ll L \rightarrow$ Couette Flow

Analytical Solution

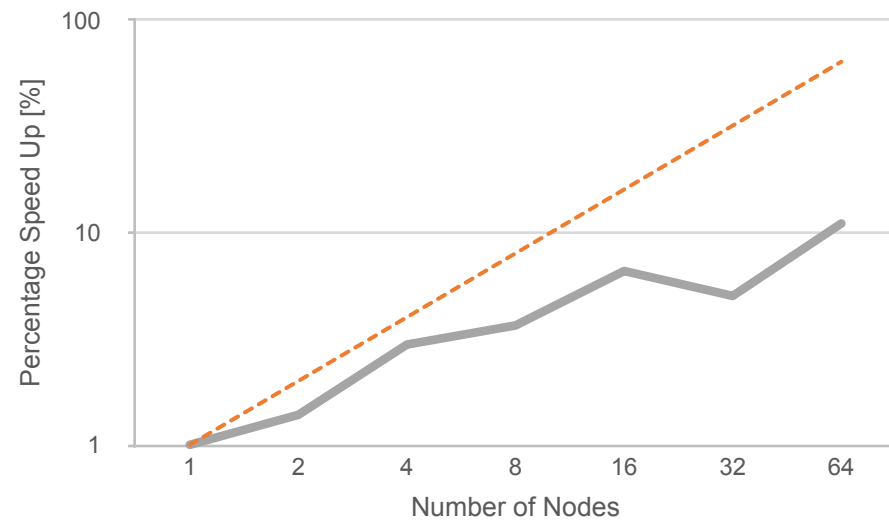
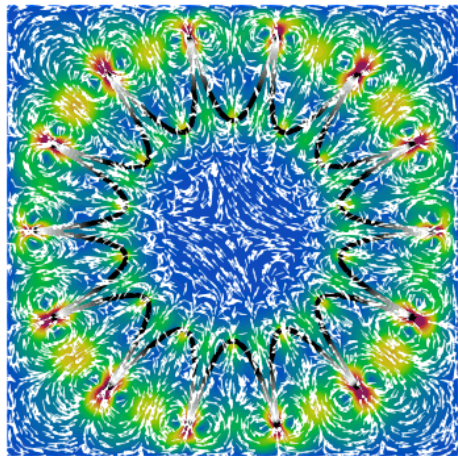
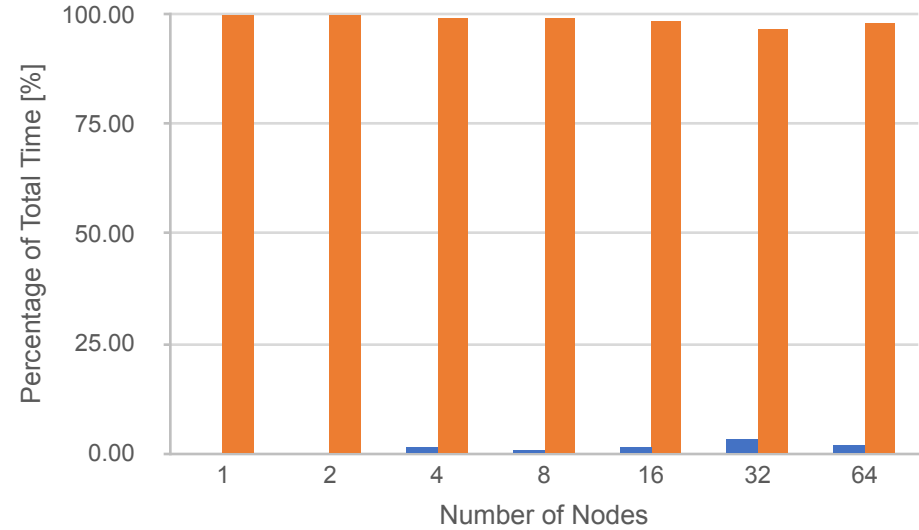
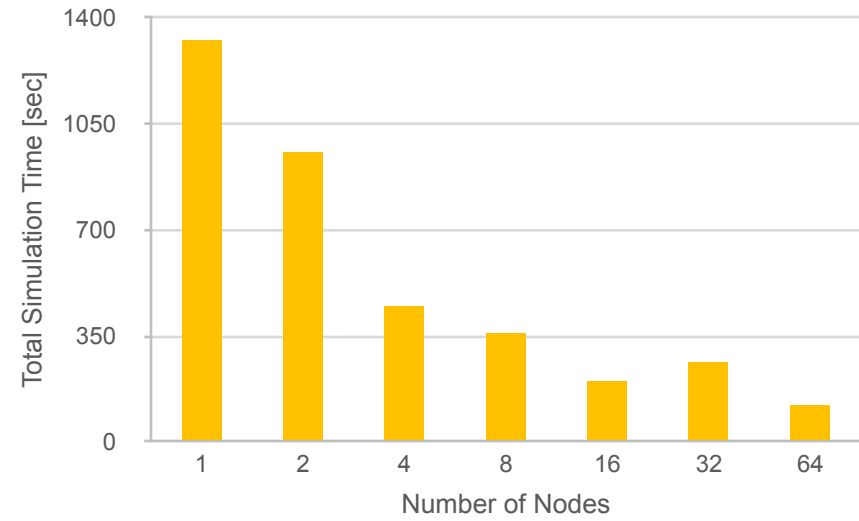
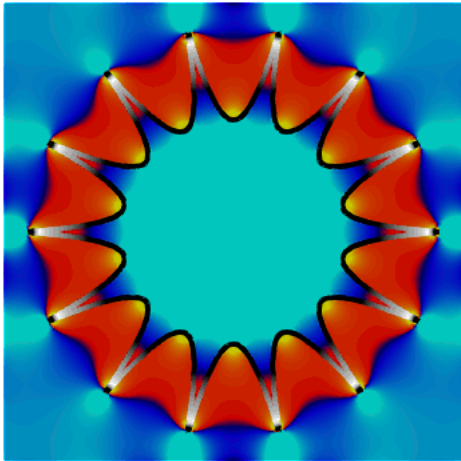
$$\delta(x,t) = \frac{F}{E}x + \frac{2F}{lE} \sum \frac{(-1)^n}{\beta^2} \exp\left(\frac{-\beta_n^2 E}{n}t\right) \sin(\beta_n^2 x), \text{ where } \eta = 2\mu/H \text{ and } \beta_n = \frac{2n-1}{2l} \pi, n \in \mathbb{N}_{>0}$$

2D Test Case Performance 1



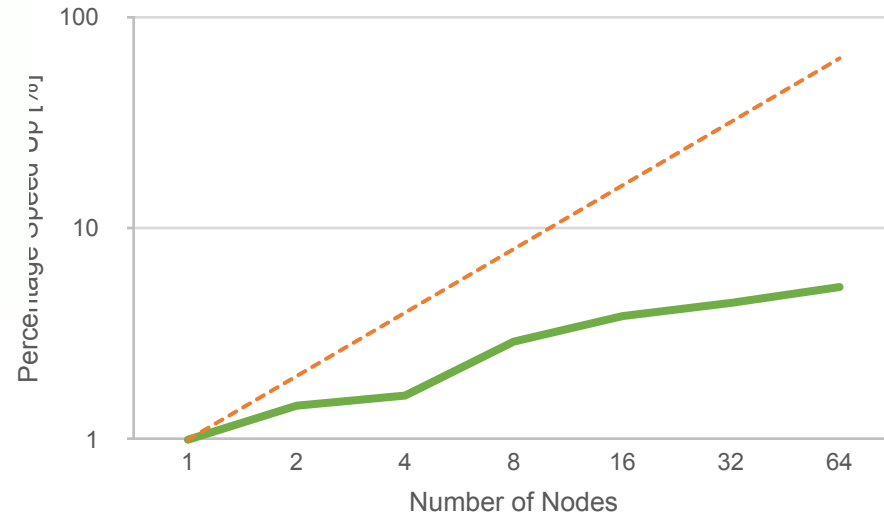
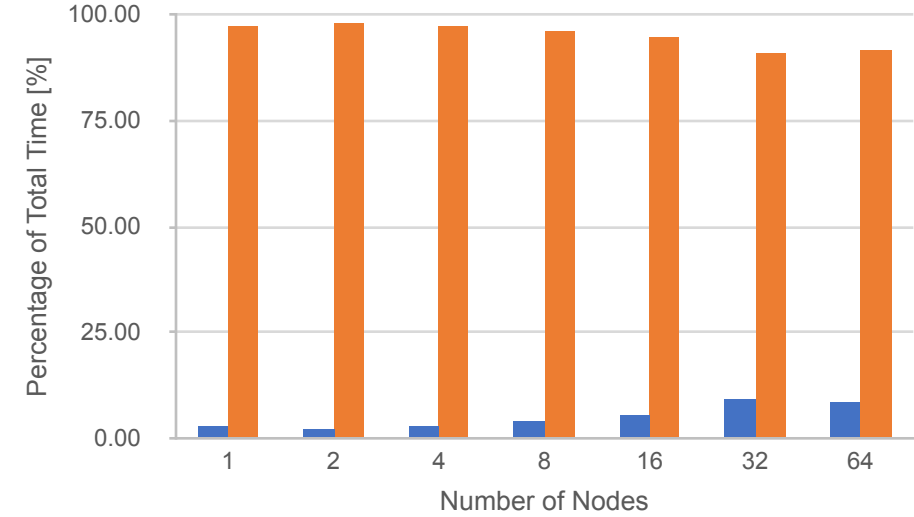
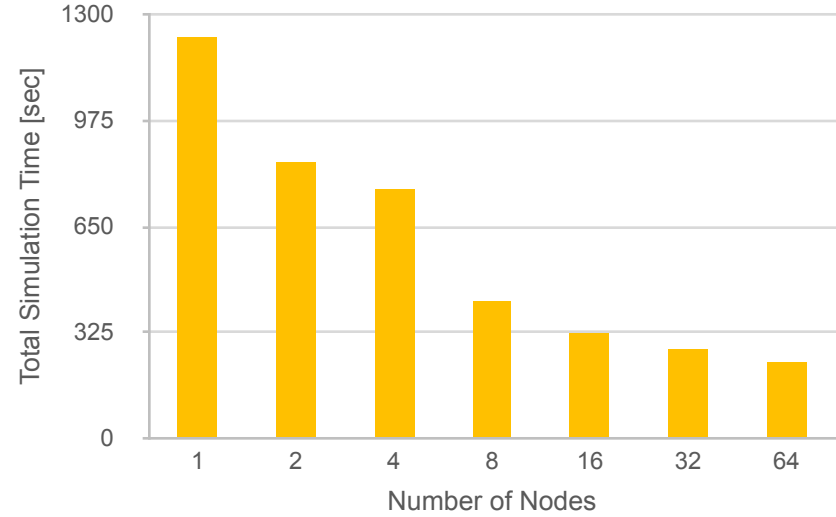
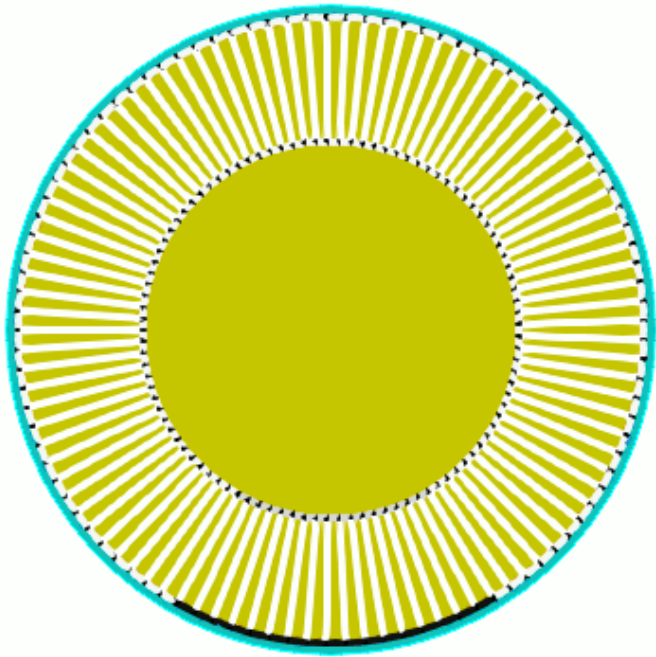
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Solid Points	166
Solid Edges	166
Fluid P2 nodes	75000
Fluid P1 nodes	18000
Fluid Elements	37000

2D Test Case Performance 2



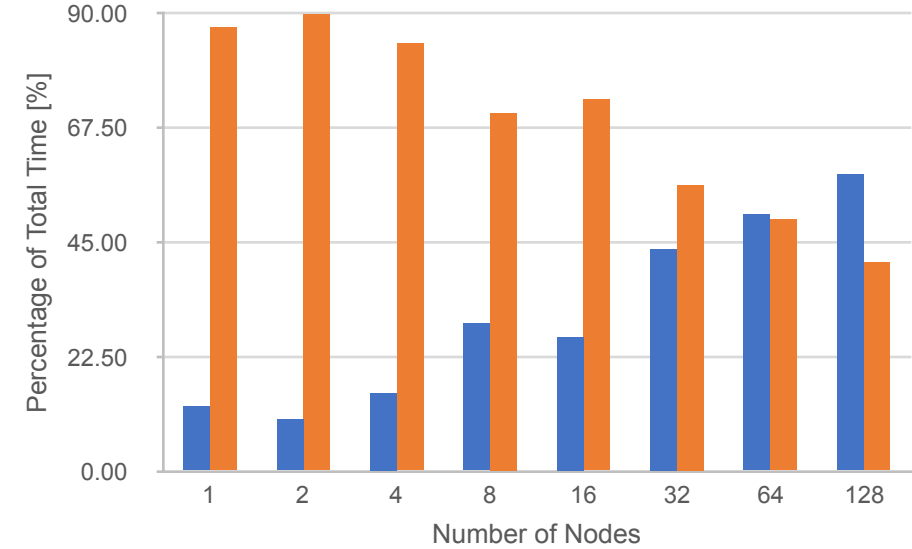
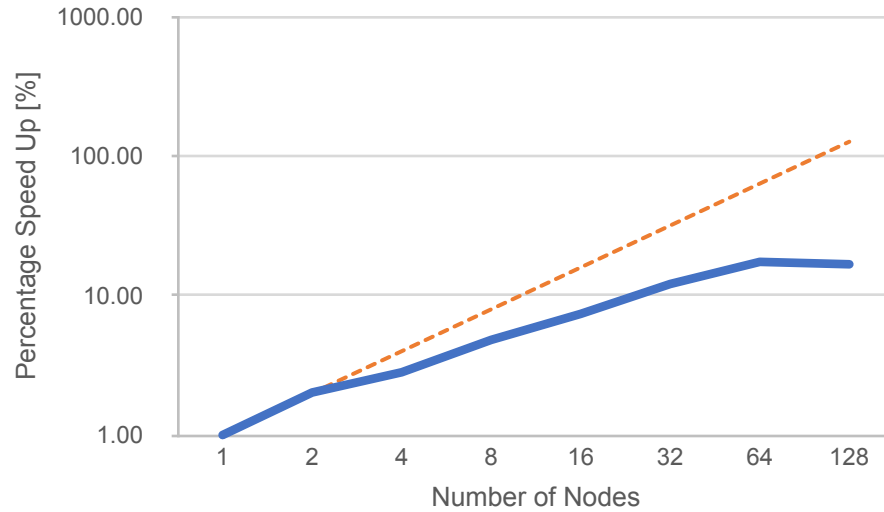
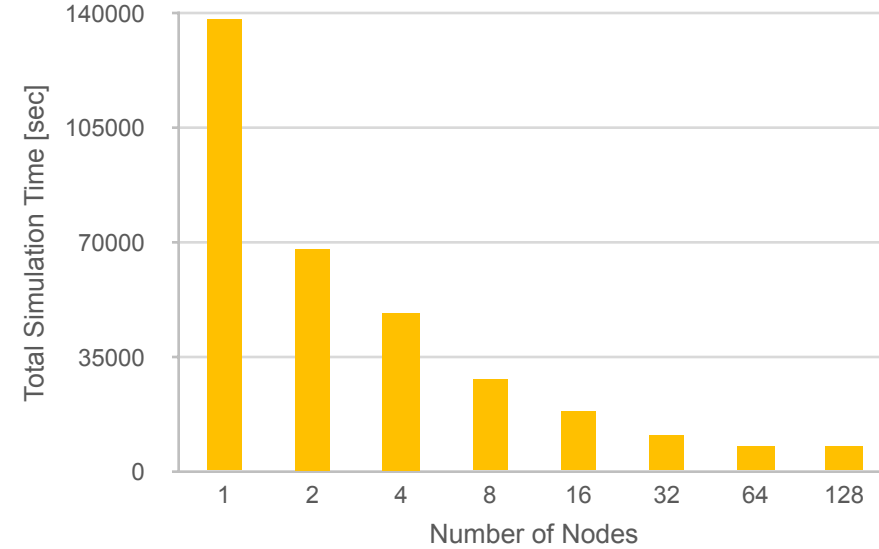
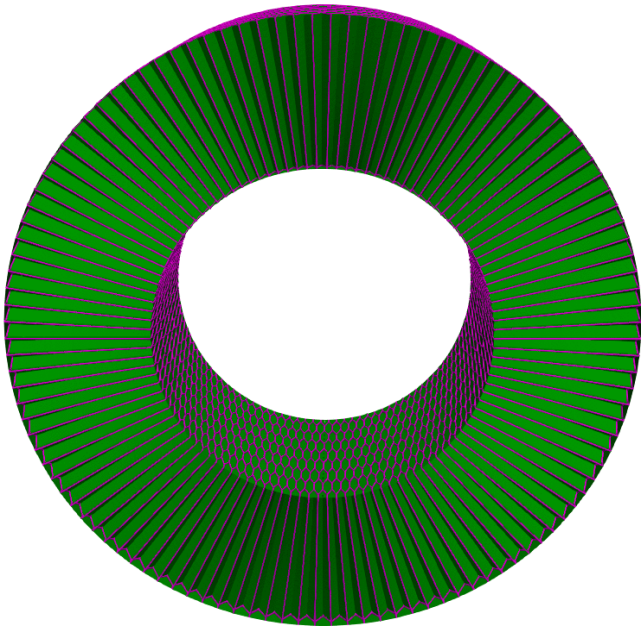
Type	Number
Solid Points	1256
Solid Edges	1256
Fluid P2 nodes	75000
Fluid P1 nodes	18000
Fluid Elements	37000

Gastrulation Case 2D



Type	Number
Solid Points	6500
Solid Edges	6600
Fluid P2 nodes	60000
Fluid P1 nodes	15000
Fluid Elements	30000

Gastrulation Case 3D



Type	Number
Solid Points	36000
Solid Lines/Triangles	180000
Fluid P1b nodes	300000
Fluid P1 nodes	53000
Fluid Elements	250000