

PETSc DMNetwork: A Library for Scalable Network PDE-Based Multiphysics Simulation

Hong Zhang

Computer Science Dept., Illinois Institute of Technology
and
Mathematics and Computer Science Div.,
Argonne National Laboratory, USA

June 2023

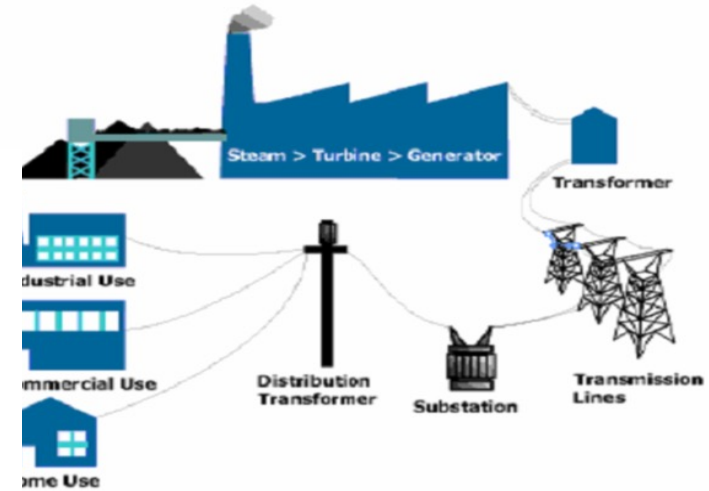
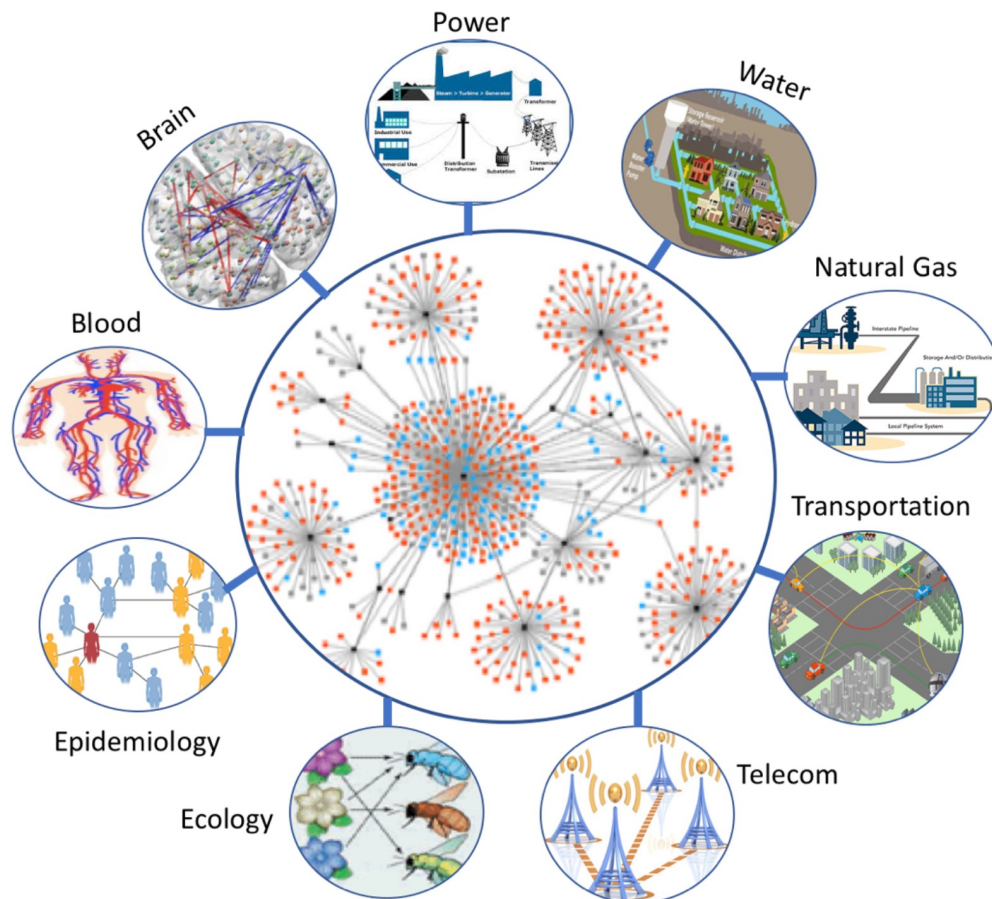


DMNNetwork

<https://www.mcs.anl.gov/petsc/dmnetwork/index.html>

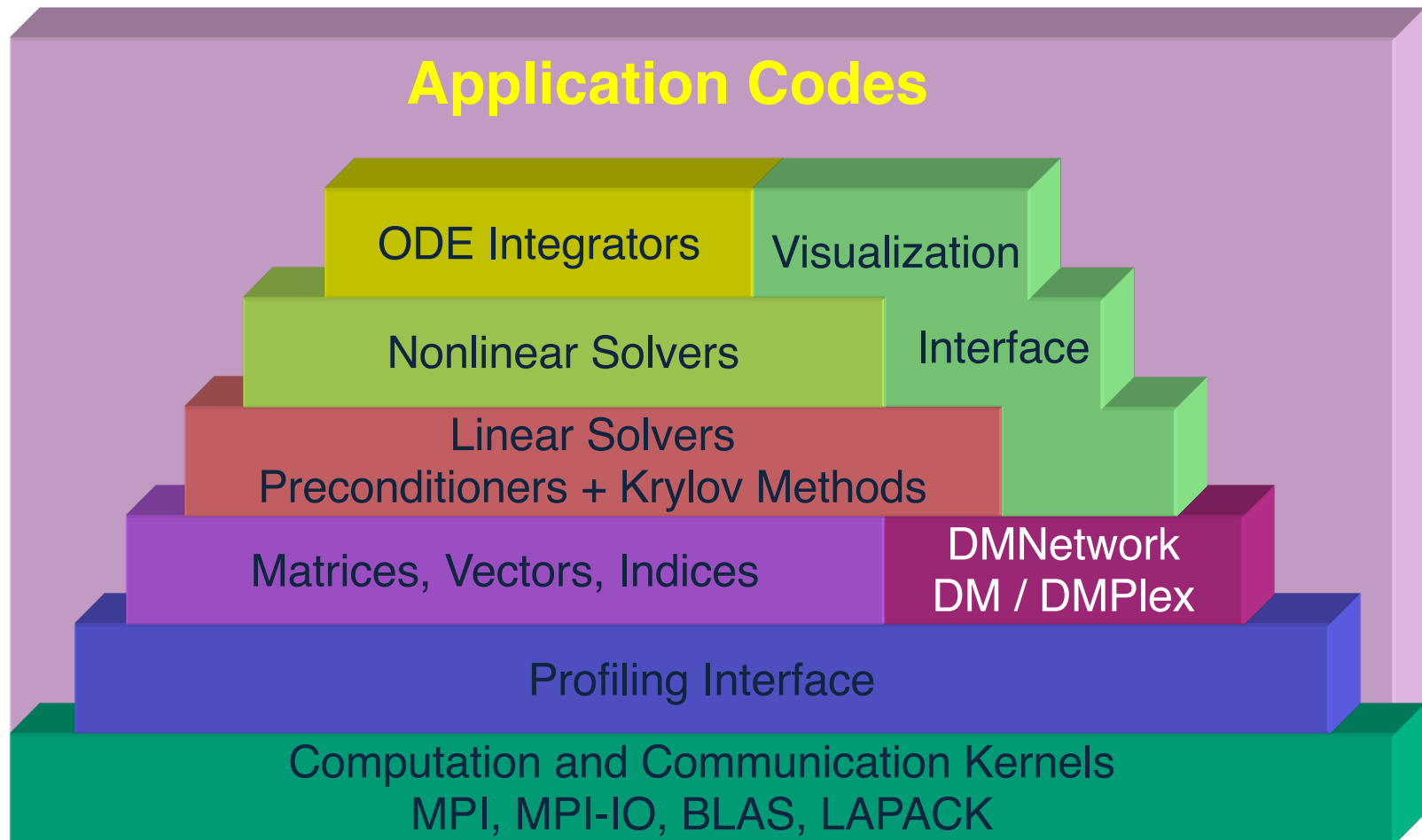
- a network multiphysics simulation package in PETSc
- data and topology management
- scalable, hierarchical and composable solvers
- eases the simulation development cycle by providing the infrastructure through simple abstractions to define and query the network components.
- [PETSc DMNetwork: A Library for Scalable Network PDE-Based Multiphysics Simulations.](#) Abhyankar S., Betrie G., Maldonado D.A, McInnes L.C., Smith B., Zhang H. , ACM Transactions on Mathematical Software, Vol. 46, No.1, Article 5, 2020.

Multiphysics Network Applications



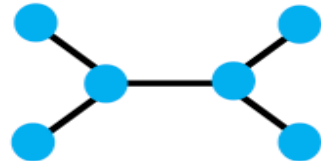
Level of
Abstraction

PETSc



DMNetwork Application Steps

Legend: ● Vertex — Edge ■ Physics Component



Create Graph

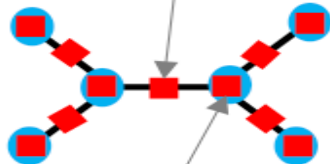


```
DMNetworkCreate()
...
DMNetworkLayoutSetup()
```

$$\frac{\partial Q}{\partial t} + gA \frac{\partial H}{\partial x} + RQ|Q| = 0$$

$$gA \frac{\partial H}{\partial t} + a^2 \frac{\partial Q}{\partial x} = 0$$

See Equations (9) - (10)



Add Physics
PDEs/AEs

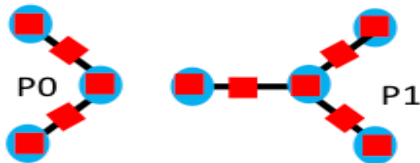


```
DMNetworkAddComponent()
DMNetworkAddNumVariables()
```

$$\sum Q_i = 0$$

$$H_i - H_j = 0$$

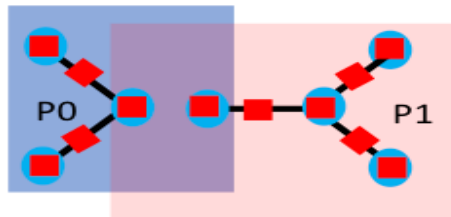
See Equations (11) - (12)



Partition



```
DMNetworkDistribute()
```



Hierarchical
Composable
Solve



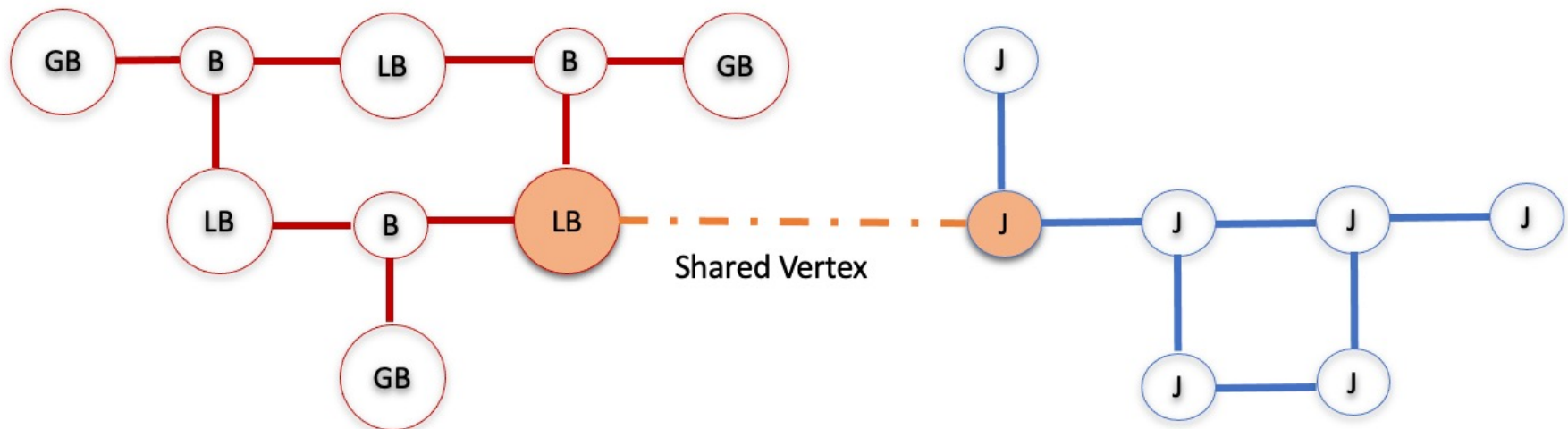
```
KSPSetDM()/SNESSetDM()/TSSetDM()
KSPSolve()/SNESsolve()/TSSolve()
```

Domain Decomposition (See Section 3)



Example: `petsc/src/snes/tutorials/network/ex1.c`

Network of Power Subnetwork, Water Subnetwork, and a Shared Vertex



Power Subnetwork:



Water Subnetwork:



Example: `petsc/src/snes/tutorials/network/ex1.c`

Let

$$X = \begin{bmatrix} X_{power} \\ X_{water} \end{bmatrix}, \quad F(X) = \begin{bmatrix} F_{power}(X_{power}) \\ F_{water}(X_{water}) \\ F_{couple}(X) \end{bmatrix}. \quad (5)$$

The nonlinear mathematical system we wish to solve is

$$F(X) = 0. \quad (6)$$

We create three nonlinear solver objects: `SNES`, `SNES_power`, and `SNES_water`. `SNES` solves the coupled Equation (6). `SNES_power` solves

$$\begin{bmatrix} F_{power}(X_{power}) \\ X_{water} - X_{water_{old}} \end{bmatrix} = 0, \quad (7)$$

and `SNES_water` solves

$$\begin{bmatrix} F_{water}(X_{water}) \\ X_{power} - X_{power_{old}} \end{bmatrix} = 0, \quad (8)$$

Example: petsc/src/snes/tutorials/network/ex1.c

```
/* Create graph */
```

```
DMNetworkCreate(comm, &dm);
```

```
DMNetworkRegisterComponent(dm, "branch", sizeof(struct_Edge_Power), &power->branch);
```

```
DMNetworkRegisterComponent(dm, "bus", sizeof(struct_VERTEX_Power), &power->bus);
```

```
DMNetworkRegisterComponent(dm, "vertex_water", sizeof(struct_VERTEX_Water), &water->vtx);
```

```
DMNetworkSetNumSubNetworks(dm, PETSC_DECIDE, Nsubnet);
```

```
DMNetworkAddSubnetwork(dm, "power", nVertices[0], nEdges[0], edgelist_power, &power_netnum);
```

```
DMNetworkAddSubnetwork(dm, "water", nVertices[1], nEdges[1], edgelist_water, &water_netnum);
```

```
DMNetworkAddSharedVertices(dm, power_netnum, water_netnum, 1, &power_svtx, &water_svtx);
```

```
DMNetworkLayoutSetUp(dm);
```



Example: petsc/src/snes/tutorials/network/ex1.c

```
/* Add components and num of variables to the power subnetwork */
DMNetworkGetSubnetwork(dm, power_netnum, &nv, &ne, &vtx, &edges);
for (i = 0; i < nv; i++) {
    DMNetworkIsSharedVertex(dm, vtx[i], &flg);
    if (flg) continue;
    DMNetworkAddComponent(dm, vtx[i], power->compkey_bus, &bus[i], nvar_power);
}
/* Add components and num of variables to the water subnetwork */
...
/* Add components and num of variables to the shred vertex */
DMNetworkGetSharedVertices(dm, &nv, &vtx);
for (i = 0; i < nv; i++) {
    DMNetworkAddComponent(dm, vtx[i], power->compkey_bus, &bus[4], nvar_power);
    DMNetworkAddComponent(dm, vtx[i], power->compkey_load, &load[0], 0)
    DMNetworkAddComponent(dm, vtx[i], water-> compkey_vtx, &vertex[0], nvar_water);
}

DMSetUp(dm);
```



Example: `petsc/src/snes/tutorials/network/ex1.c`

```
/* Partition */
DMNetworkDistribute(&dm, 0);

/* Create and setup solvers snes, snes_power and snes_water */
SNESCreate(comm, &snes);
SNESSetDM(snes, dm);
SNESSetFunction(snes, F, FormFunction, &user);
...
SNESCreate(comm, &snes_power);
SNESSetDM(snes_power, dm);
SNESSetFunction(snes_power, F, FormFunction, &user);
...
SNESCreate(comm, &snes_water);
SNESSetDM(snes_water, dm);
SNESSetFunction(snes_water, F, FormFunction, &user);
...
```



```
/* Solve */
```

```
user.it = 0; reason = SNES_DIVERGED_DTOL;  
while (user.it < it_max && reason<0) {  
    user.subsnes_id = 0;          SNESSolve(snes_power, NULL, X);  
    user.subsnes_id = 1;          SNESSolve(snes_water, NULL, X);  
    user.subsnes_id = Nsubnet; SNESSolve(snes, NULL, X));  
    user.it++;                    SNESGetConvergedReason(snes, &reason);  
};
```

Running this code with runtime options, the users can experiment with domain decomposition and solver composition/splitting at various levels of physics and computation without modifying *ex1.c*. For example, for the overall coupled system (6), we use the following options.

```
-coupled_snes_fd  
-coupled_ksp_type gmres  
-coupled_pc_type bjacobi  
-coupled_sub_pc_type lu  
-coupled_sub_pc_factor_mat_ordering_type qmd  
  
-power_pc_type asm  
-power_sub_pc_type lu  
-power_sub_pc_factor_mat_ordering_type qmd
```



```
/* Function evaluation */
```

```
FormFunction(SNES snes, Vec X, Vec F, void *appctx) {
```

```
...
```

```
SNESGetDM(snes, &dm);
```

```
DMGetLocalVector(dm, &localX); DMGetLocalVector(dm, &localF);
```

```
DMGlobalToLocalBegin(dm, X,..., localX); DMGlobalToLocalEnd(dm, X,..., localX);
```

```
/* Function evaluation on power subnetwork */
```

```
DMNetworkGetSubnetwork(dm, 0, &nv, &ne, &vtx, &edges);
```

```
if (user->subsnes_id == 1) { /* snes_water only */
```

```
FormFunction_Dummy(dm, localX, localF, nv, ne, vtx, edges, user); /* localF(X) = localX - localXold */
```

```
} else {
```

```
FormFunction_Power(dm, localX, localF, nv, ne, vtx, edges, &appctx_power);
```

```
}
```

```
/* Function evaluation on water subnetwork */
```

```
...
```

```
/* Function evaluation at shared vertex */
```

```
DMNetworkGetSharedVertices(dm, &nv, &vtx);
```

```
...
```

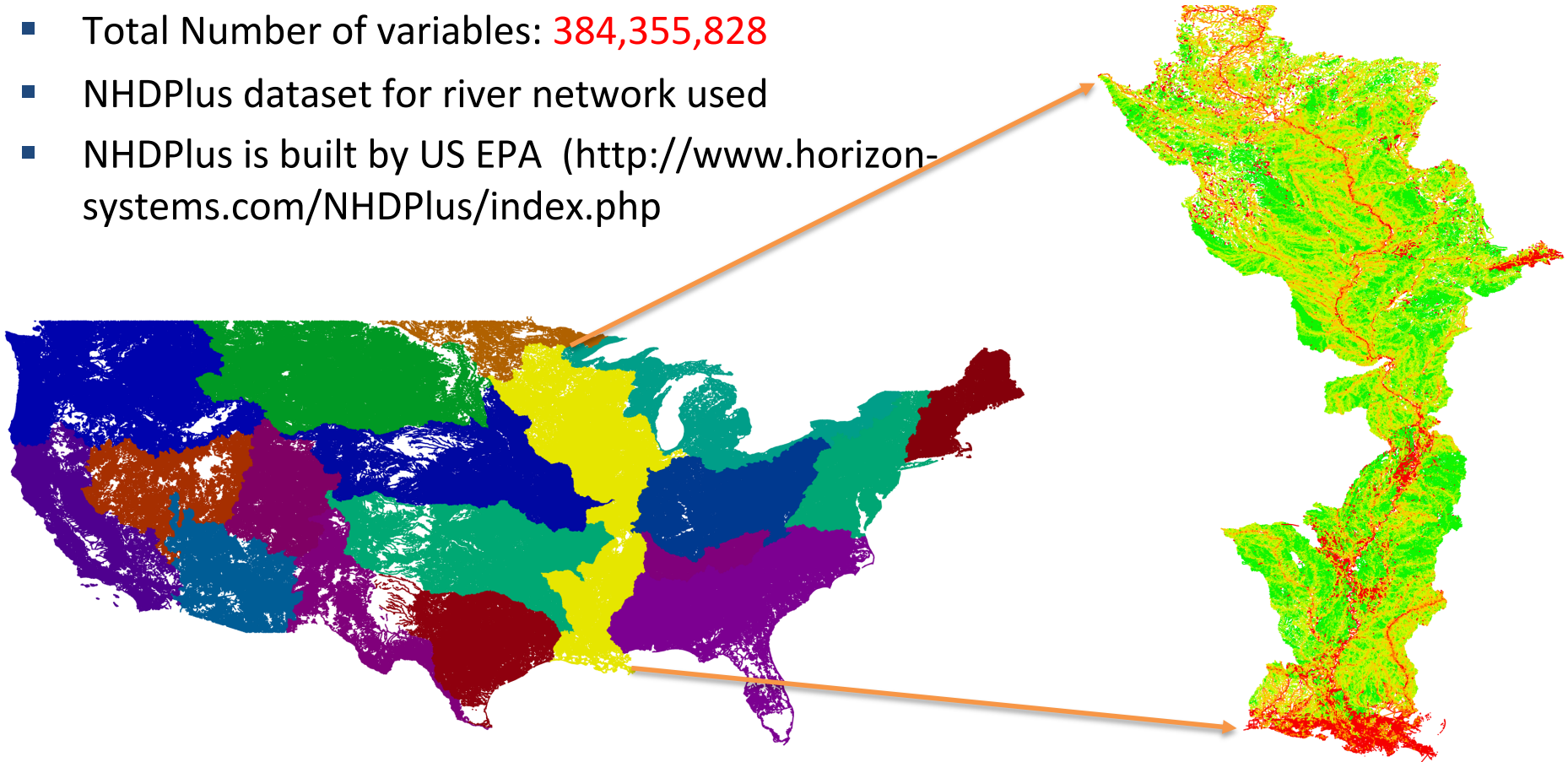
```
}
```



Continental US Major Rivers Simulation

(G. Betrie, H. Zhang)

- 3,098,638 reaches and 3,028,968 junctions
- Total Number of variables: 384,355,828
- NHDPlus dataset for river network used
- NHDPlus is built by US EPA (<http://www.horizon-systems.com/NHDPlus/index.php>)



PETSc DMNetwork: A Library for Scalable Network PDE-Based Multiphysics Simulations, Abhyankar S., Betrie G., Maldonado D.A, McInnes L.C., Smith B., Zhang H. , ACM Transactions on Mathematical Software, 2020.

Parallel Primal-Dual Interior Point Method for Dynamic Optimal Power Flow

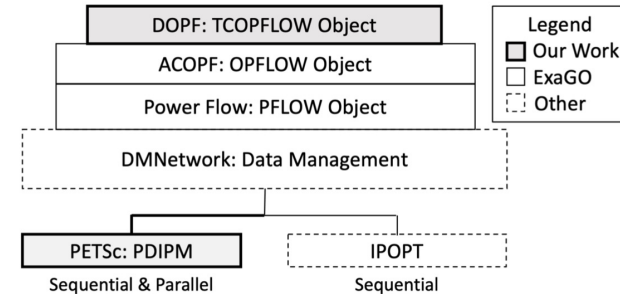
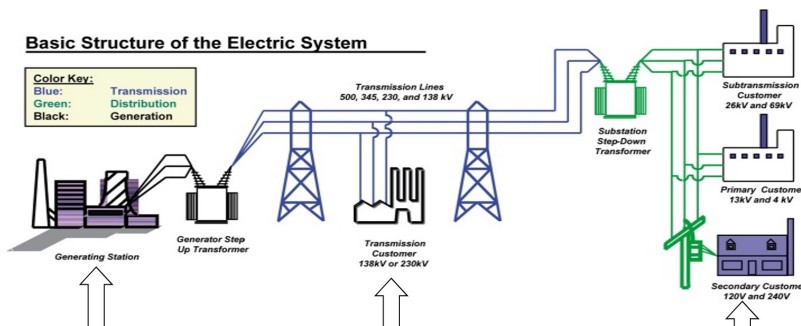
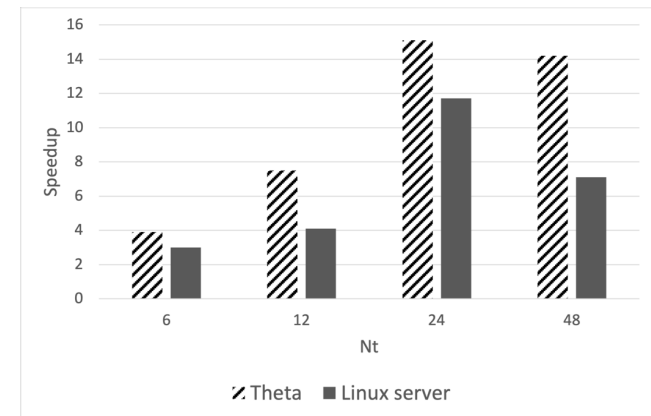


Fig. 2: Software structure of DOPF within ExaGO.

- Developed a new parallel primal-dual interior point solver in TAO
- Used multiphysics networks to exploit problem structure with a blocked preconditioner
- Applied to multiperiod optimal power flow networks with computational ease and efficiency
- Collaboration with PNNL for power-grid modeling, including members of the GridPACK and ECP ExaSGD projects

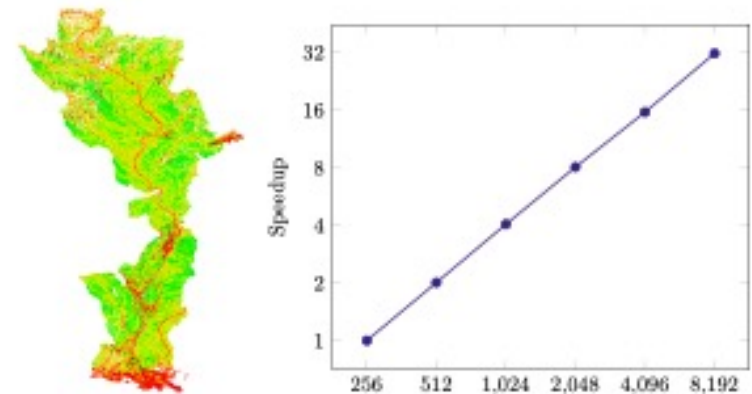
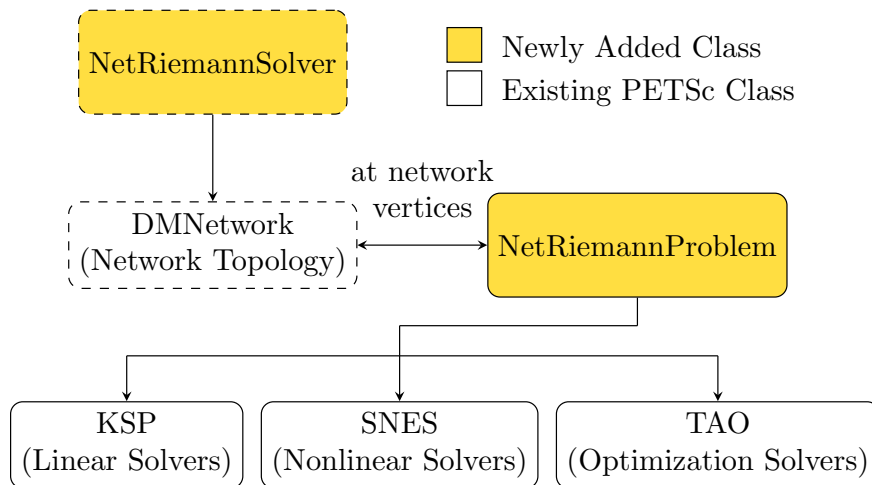


Parallel Primal-Dual Interior Point Method for the Solution of Dynamic Optimal Power Flow,
 R. Sundermann, S. Abhyankar, H. Zhang, J. Kimn, T. Hansen, 2022, IET Generation, Transmission & Distribution.

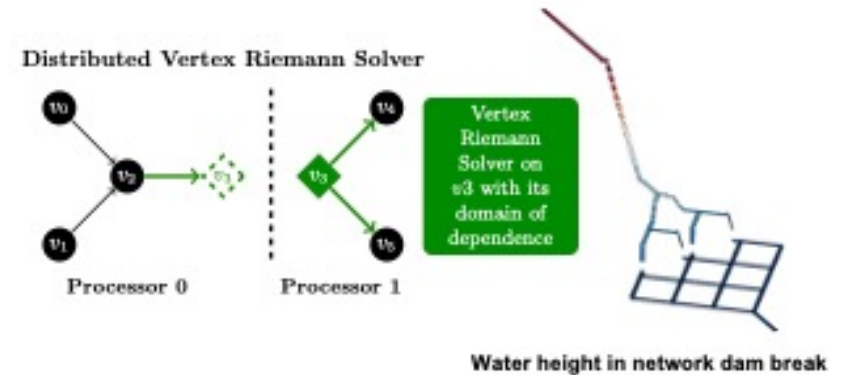


Scalable Riemann Solvers For Hyperbolic Network Simulations

Scalable vertex Riemann solvers for the simulation of hyperbolic conservation laws on networks, using discontinuous Galerkin discretization for network edges



Scalability of a Mississippi River network simulation.



A. Hamilton, J. Qiu, H. Zhang, "Scalable Riemann Solvers with the Discontinuous Galerkin Method for Hyperbolic Network Simulations," Proceedings of the PASC Conference, 2023.





Traffic Flow: Data-Intensive Computation

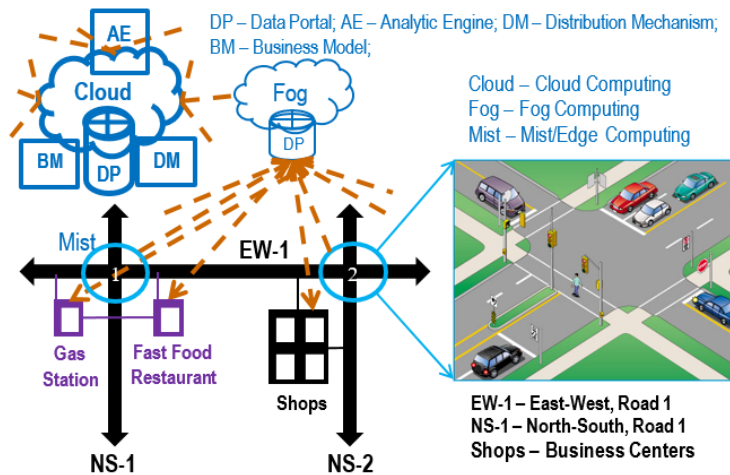


Figure 1: A representative GCI system

$$\text{Total Profit: } P_{T-t} = P_{\text{Sale-t}} + P_{\text{Staffing-t}} + P_{\text{Waste-t}}$$

Approaches:

- 1) OpenStreetMap and Sumo:
- 2) Google Maps Platform:
- 3) Location IQ

- Geospatial Cyberinfrastructure for Regional Economic Growth
- Exploring new capabilities motivated by the needs of data-driven computing.
- Collaboration with Prof. [A. Asaduzzaman](#) of WSU via SHI SRP-HPC PROGRAM
- Student participants: [D. Campbell](#), [N. Nawal](#).



New addition: PetscViewer with CSV format

-- requires matplotlib and pandas
(D. Campbell, A. Hamilton, H. Zhang)

Example: `petsc/src/snes/tutorials/network/ex1.c`

```
mpiexec -n 3 ./ex1  
-petscpartitioner_type parmetis  
-dmnetwork_view draw  
-dmnetwork_view_distributed draw  
-dmnetwork_view_all_ranks
```

```
./ex1 -monitorIteration -monitorColor  
-power_snes_max_it 0  
-water_snes_max_it 0  
-coupled_snes_max_it 10  
-draw_pause 5.0
```